1^{ST} ORDER CIRCUITS

RL OR RC

GOVERNED BY 1^{ST} ORDER D.E.

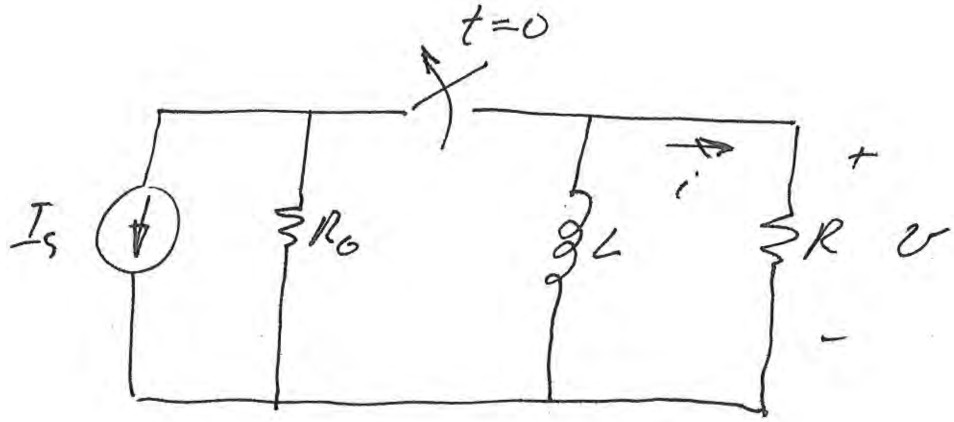
THREE ANALYSIS PHASES:

1) NATURAL RESPONSE (ENERGY STORED IN INDUCTOR OR CAPACITOR IS RELEASED TO A RESISTIVE NETWORK)

2) STEP RESPONSE (VOLTAGE OR CURRENT SOURCE SUDDENLY APPLIED TO REACTIVE NETWORK)

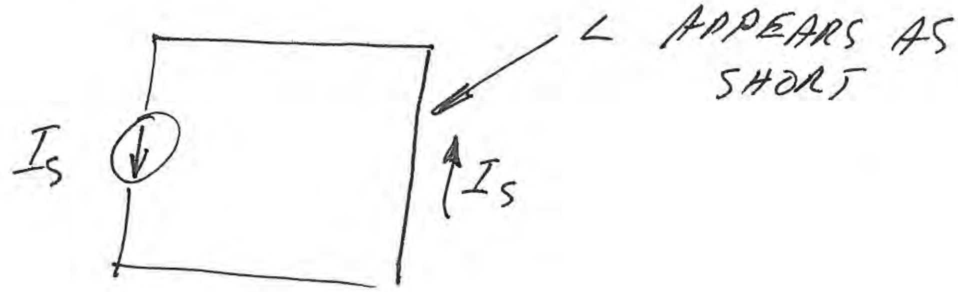
3) GENERAL METHOD

NATURAL RESPONSE
(INTRINSIC TO CIRCUIT)

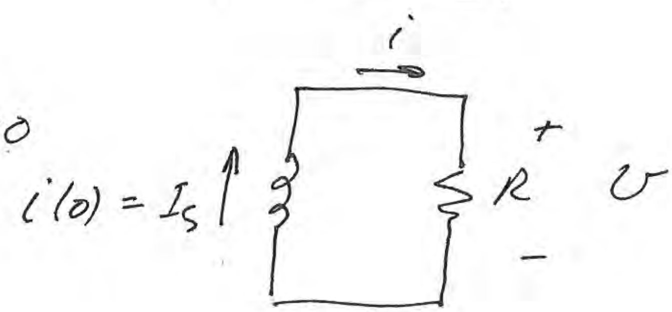


SWITCH CLOSED LONG TIME BEFORE $t=0$

$t<0$



$t\geq 0$



ANALYSIS BY KVL:

$$L\frac{di}{dt} + Ri = 0 \qquad (1^{ST}\ \text{ORDER D.E.})$$

$$\frac{di}{dt} = -\frac{R}{L} i$$

$$\frac{di}{i} = -\frac{R}{L} dt$$

$$\frac{dx}{x} = -\frac{R}{L} dy$$

$$\int_{i(t_0)}^{i(t)} \frac{dx}{x} = -\frac{R}{L} \int_{t_0}^{t} dy$$

$$\ln x \Big|_{i(t_0)}^{i(t)} = -\frac{R}{L}(t - t_0)$$

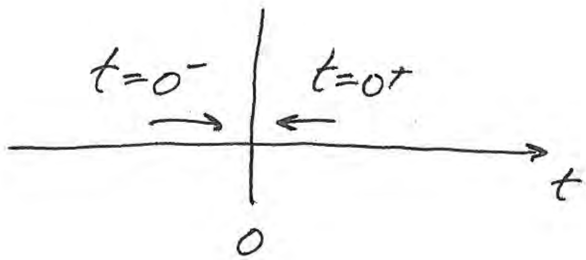
$$\ln \frac{i(t)}{i(t_0)} = -\frac{R}{L}(t - t_0)$$

$$\frac{i(t)}{i(t_0)} = e^{-\frac{R}{L}(t-t_0)}$$

$$i(t) = i(t_0) e^{-\frac{R}{L}(t-t_0)}$$

FOR THIS PROBLEM, $t_0 = 0$

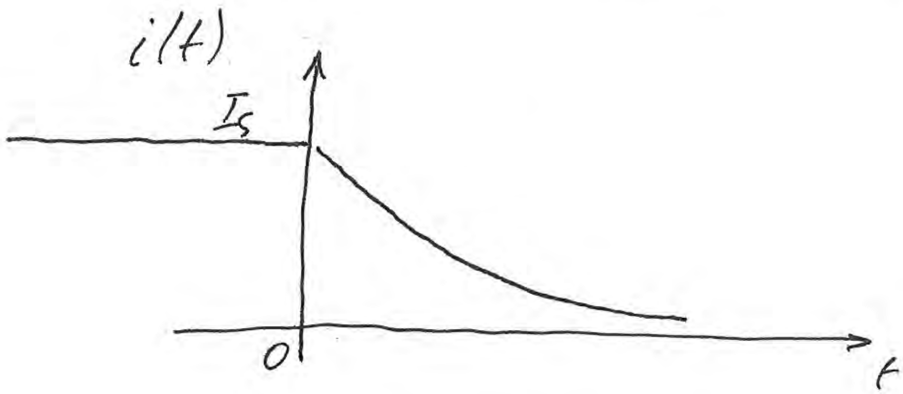
$$i(t) = i(0)\, e^{-\frac{R}{L}t}$$



CURRENT THROUGH INDUCTOR
CANNOT CHANGE INSTANTANEOUSLY

$\therefore \quad i(0^-) = i(0^+) = I_s$

FINALLY $\quad i(t) = I_s\, e^{-\frac{R}{L}t} \quad ; \quad t \geq 0$



WHAT ABOUT VOLTAGE, v ?

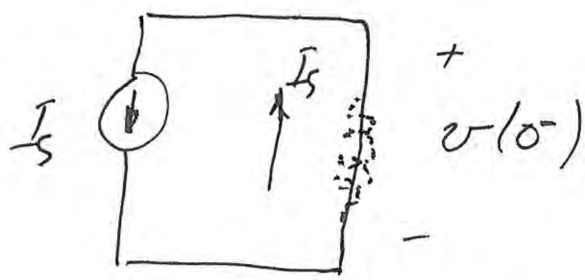
$$v(t) = i(t)R$$

$$v(t) = I_s R e^{-\frac{R}{L}t}$$

CAREFUL...

WHAT IS DOMAIN OF THIS SOLUTION?

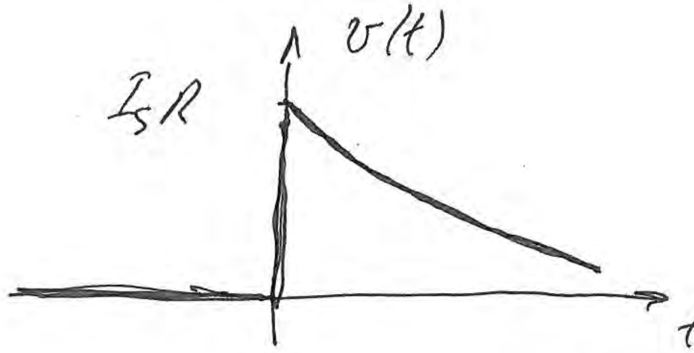
RECALL $t<0$



BECAUSE INDUCTOR IS SHORT, $v(0^-) = 0$

FROM SOLUTION, $v(0^+) = I_s R$

VOLTAGE CHANGES INSTANTANEOUSLY AT $t=0$



$\therefore \quad v(t) = I_s R e^{-\frac{R}{L}t} \; ; \; t \geq 0^+$

POWER DISSIPATED IN RESISTOR

$$p = vi = i^2 R = \frac{v^2}{R}$$

$$p = I_s^2 R e^{-2\frac{R}{L}t} \quad ; \; t \geq 0^+$$

ENERGY DELIVERED TO RESISTOR

$$w = \int_0^t p(x)\,dx = \int_0^t I_s^2 R e^{-2\frac{R}{L}x}\,dx$$

$$= I_s^2 R \left(-\frac{1}{2\,R/L} e^{-2\frac{R}{L}x} \Big|_0^t \right)$$

$$= \frac{1}{2} I_s^2 L \left(1 - e^{-2\frac{R}{L}t} \right) \quad ; \; t \geq 0$$

WHAT IS ENERGY INITIALLY STORED IN INDUCTOR ?

$$i(t) = I_s e^{-\frac{R}{L}t} \; ; \; t \geq 0$$

DIMENSIONLESS

$$\frac{[R]}{[L]} = \frac{\Omega}{H} = \frac{\Omega}{V \cdot S/A} = \cancel{\#\#}$$

$$= \frac{V/A}{V \cdot S/A}$$

$$= \frac{1}{S}$$

DENOTE $\frac{L}{R} \equiv \tau$

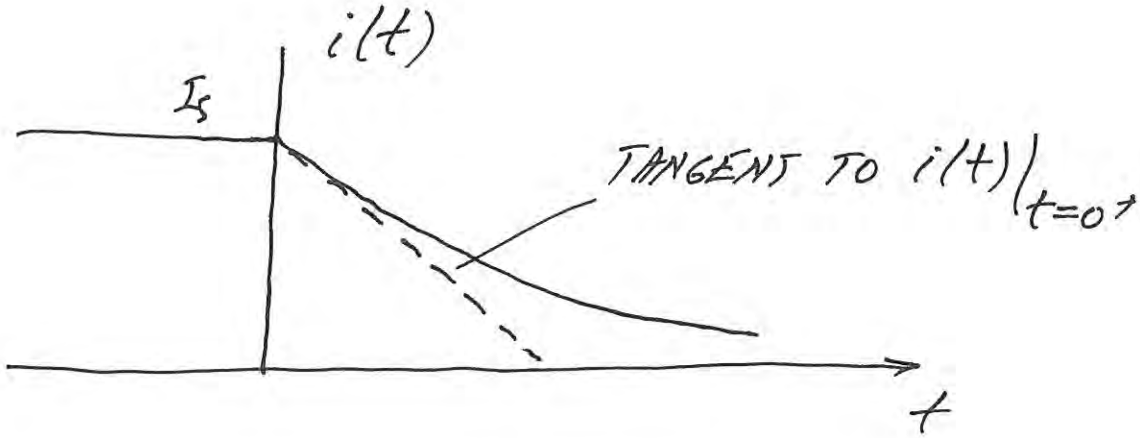
$\tau \equiv$ TIME CONSTANT

$[\tau] = S$

$$i(t) = I_s e^{-t/\tau} \; ; \; t \geq 0$$

INSPECT $\left.\frac{d}{dt} i(t)\right|_{t=0^+}$

$$\left. -\frac{I_s}{\tau} e^{-t/\tau} \right|_{t=0^+} = -I_s/\tau$$



EQ OF TANGENT LINE: $mt + b = i$

$$i = I_s - \frac{I_s}{\tau} t$$

THIS TANGENT LINE $= 0$ WHEN $t = \tau$

$$i(t) = I_s e^{-t/\tau} \; ; \; t \geq 0$$

TRANSIENT SIGNAL EFFECTIVELY ZERO

BEYOND $t \sim 5\tau$

SOLUTION STRATEGY

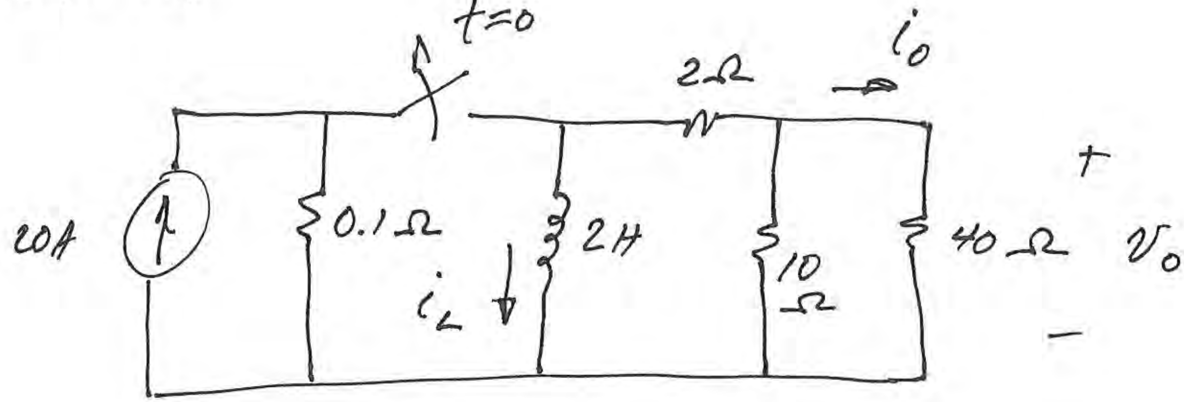
1) DETERMINE INITIAL CURRENT

2) DETERMINE TIME CONSTANT

3) USE FORM

$$i(t) = i(0) e^{-t/\tau}$$

EX 7.1



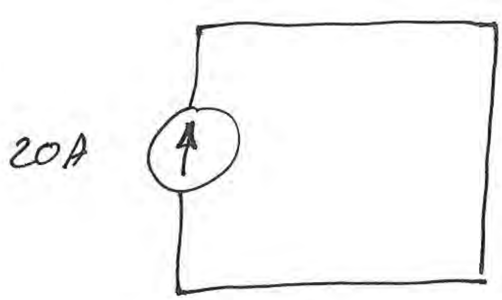
WANT $i_L(t)$; $t \geq 0$

$i_o(t)$; $t \geq 0^+$

$v_o(t)$; $t \geq 0^+$

ENERGY DISSIPATED IN 10 Ω RESISTOR

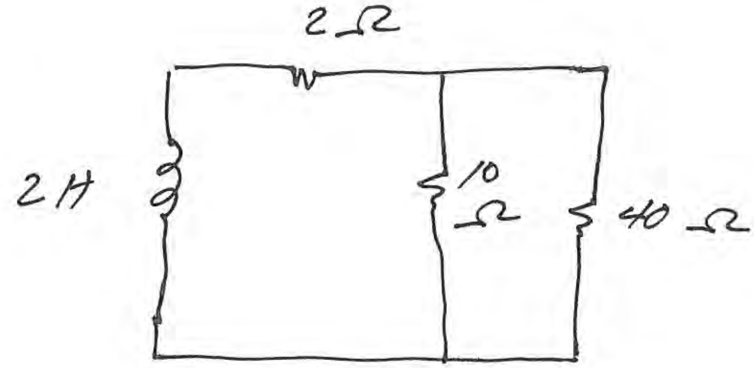
1) ESTABLISH INITIAL CURRENT



$i_L(0^-) = 20A = i_L(0^+)$

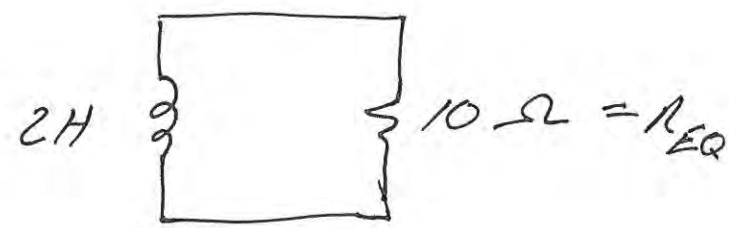
(CURRENT IN INDUCTOR CANNOT CHANGE INSTANTANEOUSLY)

$t < 0$



$t > 0$

2) DETERMINE TIME CONSTANT

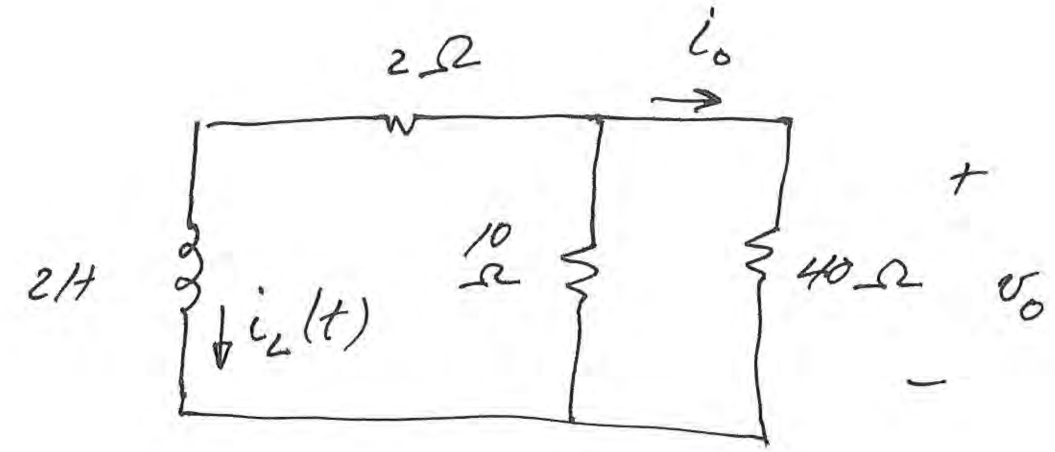


$$\tau = L/R_{EQ} = 0.2 \text{ SEC}$$

3) $i_L(t) = 20\,e^{-5t}\ A\ ;\ t \geq 0$

NEED NOT DISTINGUISH BETWEEN 0^- & 0^+

WHY?



i_o BY CURRENT DIVISION

$$i_o = -i_L \frac{10}{10+40}$$

$$i_o(t) = -4e^{-5t}\,A\,;\ t \geq 0^+$$

WHY NOT JUST

$t \geq 0$

?

VOLTAGE, v_o, BY OHM'S LAW:

$$v_o(t) = 40\,i_o(t) = -160e^{-5t}\,V;\ t \geq 0^+$$

POWER DISSIPATED IN 10 Ω RESISTOR:

$$P_{100\Omega}(t) = \frac{v_0^2(t)}{10\,\Omega} = 2560\,e^{-10t}\ \text{W}$$

$$t \geq 0^+$$

TOTAL ENERGY DISSIPATED

$$W_{100\Omega} = \int_0^{\infty} P_{100\Omega}(t)\,dt$$

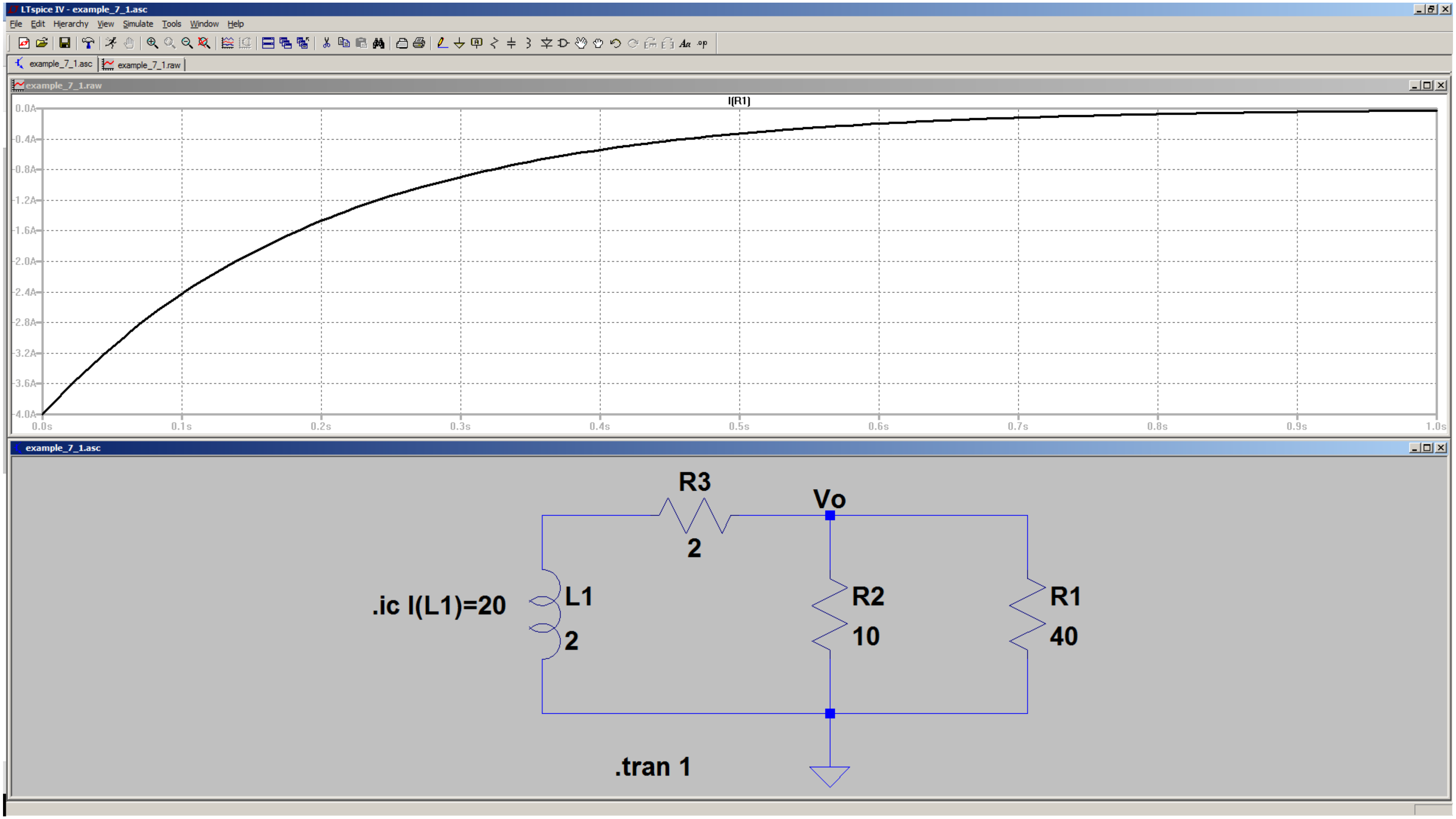
$$= \int_0^{\infty} 2560\,e^{-10t}\,dt$$

$$= 2560\left(-\frac{1}{10}e^{-10t}\Big|_0^{\infty}\right)$$

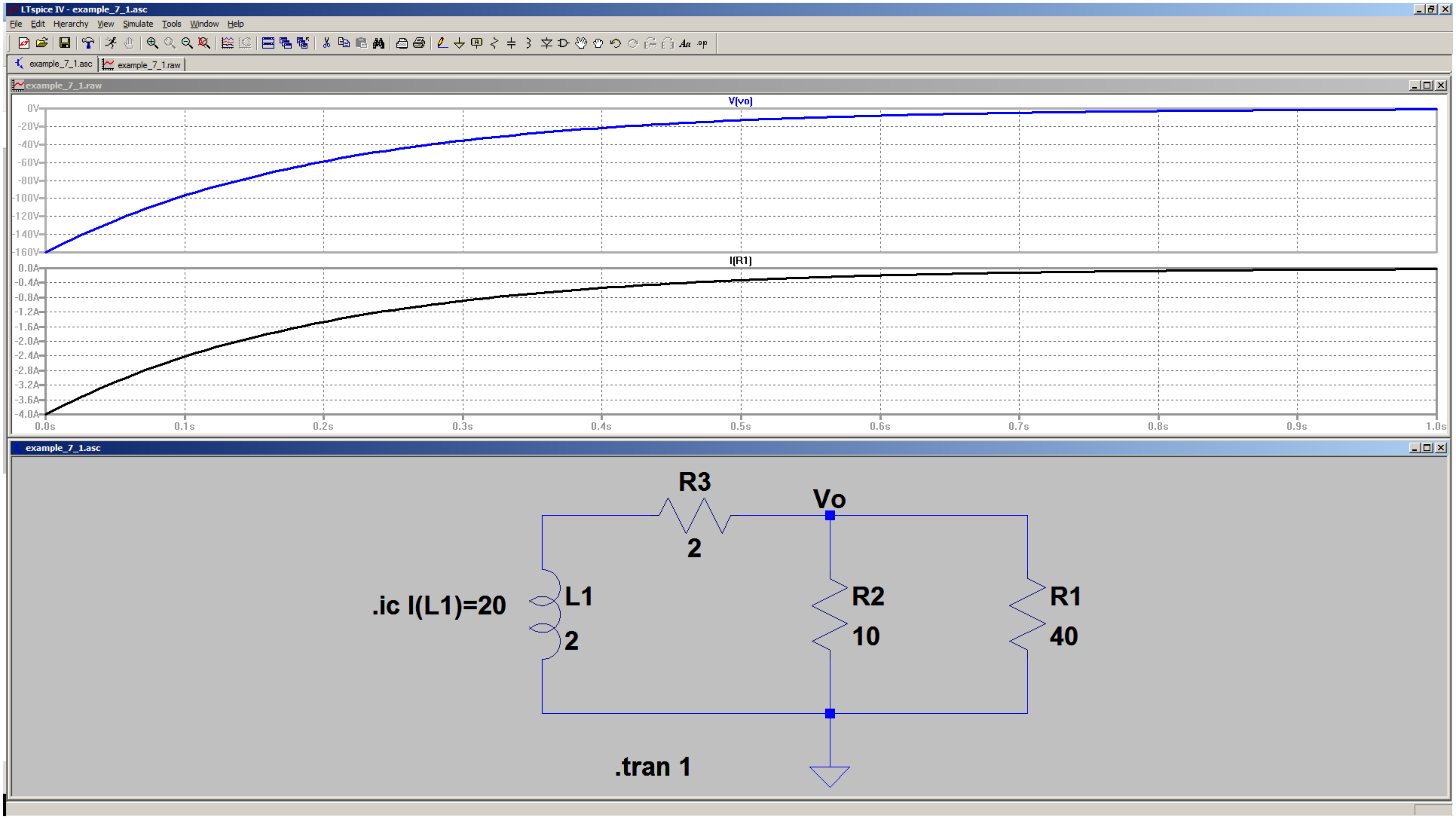
$$W_{100\Omega} = 256\ \text{J}$$

INITIAL ENERGY IN INDUCTOR IS

$$W_L(0) = \frac{1}{2}L\,i^2(0) = \frac{1}{2}\,2(20)^2 = 400\ \text{J}$$

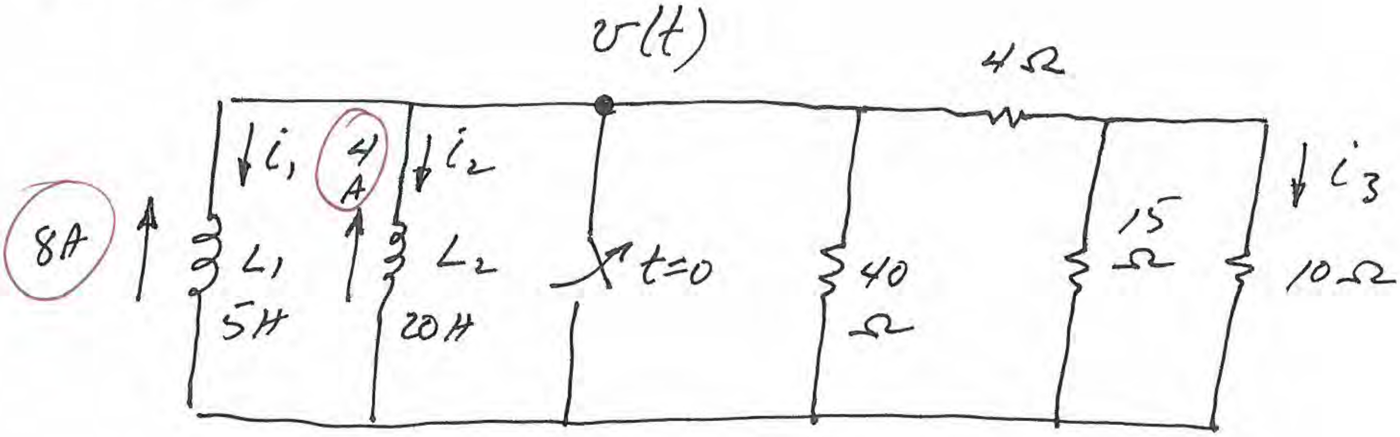


LTspice IV - example_7_1.asc
File Edit Hierarchy View Simulate Tools Window Help
example_7_1.asc
example_7_1.raw
I(R1)
0.0A
-0.4A
-0.8A
-1.2A
-1.6A
-2.0A
-2.4A
-2.8A
-3.2A
-3.6A
-4.0A
0.0s
0.1s
0.2s
0.3s
0.4s
0.5s
0.6s
0.7s
0.8s
0.9s
1.0s
R3
2
Vo
.ic I(L1)=20
L1
2
R2
10
R1
40
.tran 1



LTspice IV - example_7_1.asc
File Edit Hierarchy View Simulate Tools Window Help
example_7_1.asc
example_7_1.raw
V(vo)
0V
-20V
-40V
-60V
-80V
-100V
-120V
-140V
-160V
I(R1)
0.0A
-0.4A
-0.8A
-1.2A
-1.6A
-2.0A
-2.4A
-2.8A
-3.2A
-3.6A
-4.0A
0.0s
0.1s
0.2s
0.3s
0.4s
0.5s
0.6s
0.7s
0.8s
0.9s
1.0s
R3
2
Vo
.ic I(L1)=20
L1
2
R2
10
R1
40
.tran 1

EX 7.2



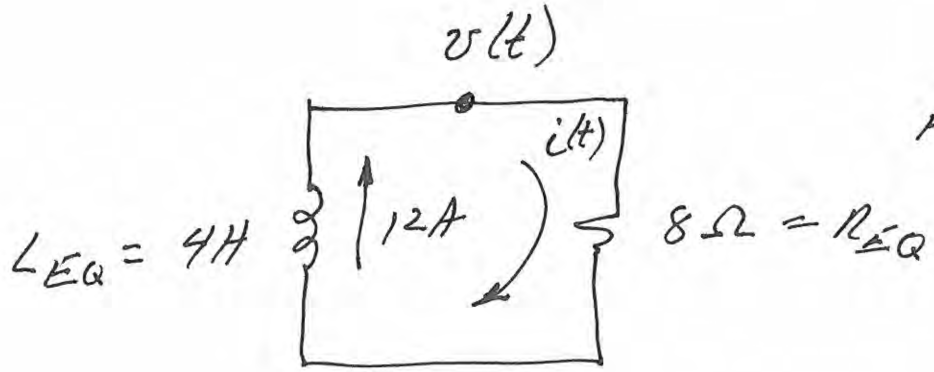
INITIAL CURRENTS

WANT: $i_1(t)$, $i_2(t)$, $i_3(t)$ FOR $t \geq 0$

INITIAL ENERGY IN INDUCTORS

ENERGY IN INDUCTORS AS $t \to \infty$

DEMONSTRATE WHERE THIS ENERGY GOES



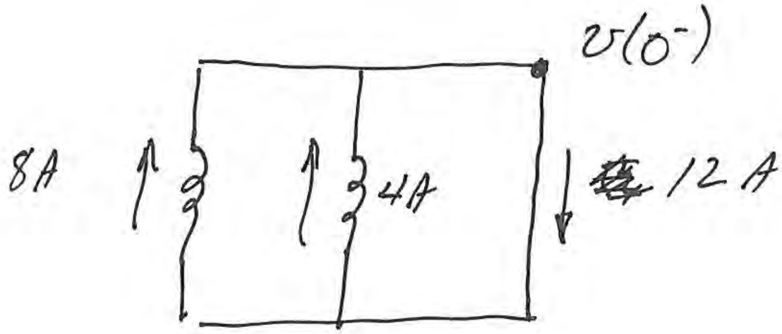
KEY IS FINDING $v(t)$

IMMEDIATELY, $i(t) = 12e^{-2t}$ A ; $t \geq 0$

BY OHM'S LAW,

$$v(t) = 8\,i(t) = 96e^{-2t}\ \text{V}\ ;\ t \geq 0^+$$

WHY 0^+ ?



$t < 0$

$$i_1(t) = \frac{1}{L_1}\int_0^t v(x)\,dx + i_1(0)$$

$$i_1(t) = \frac{1}{5}\int_0^t 96e^{-2x}\,dx - 8$$

$$= \frac{96}{5}\left(-\frac{1}{2}e^{-2x}\Big|_0^t\right) - 8$$

$$= \frac{48}{5}\left(1 - e^{-2t}\right) - 8\ \text{A}$$

$$i_1(t) = 1.6 - 9.6e^{-2t}\ A \ ; t \geq 0$$

SIMILARLY

$$i_2(t) = -1.6 - 2.4e^{-2t}\ A \ ; t \geq 0$$

$$i_1(t)\Big|_{t\to\infty} = 1.6\,A$$

$$i_2(t)\Big|_{t\to\infty} = -1.6\,A$$

$$w_{L_1}(0) = \frac{1}{2} L_1 i_1^2(0) = \frac{1}{2}\, 5\,(8)^2 = 160\,J$$

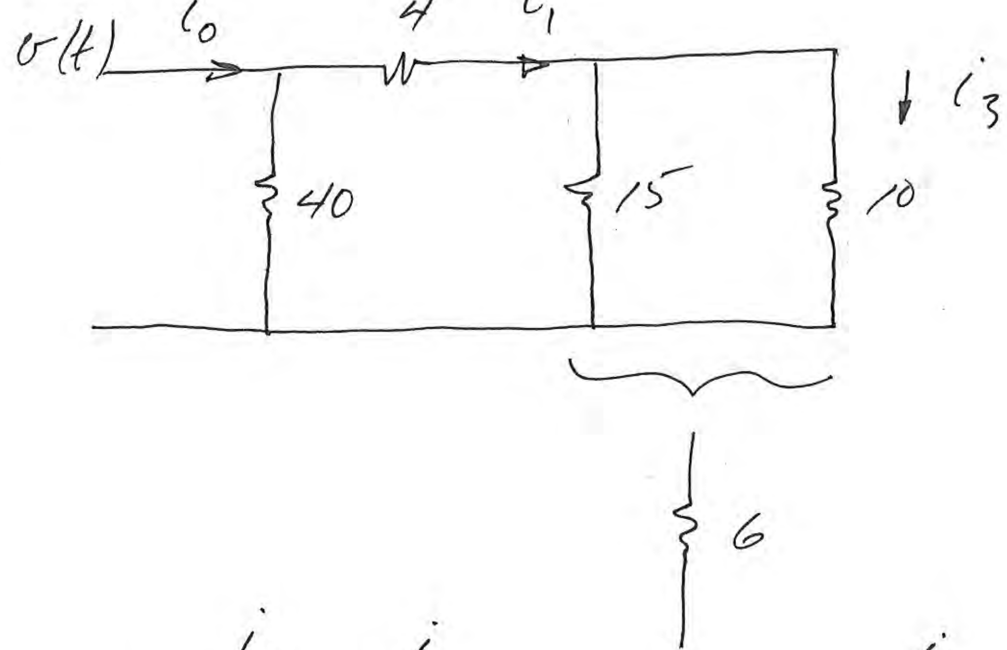
$$w_{L_2}(0) = \frac{1}{2} L_2 i_2^2(0) = \frac{1}{2}\, 20\,(4)^2 = 160\,J$$

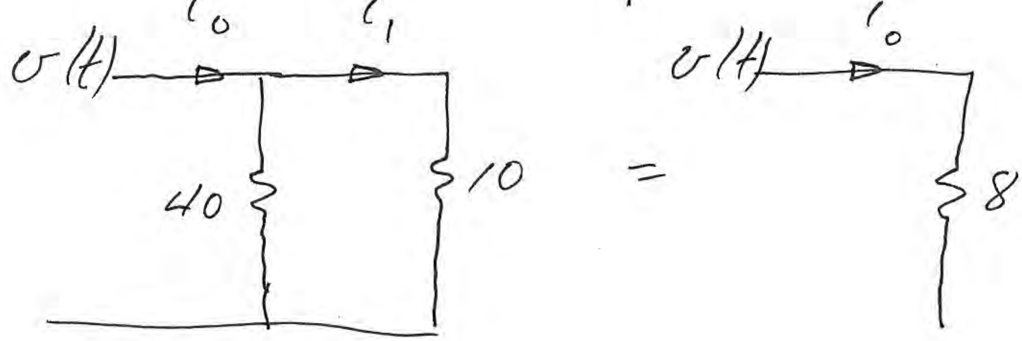
$$w_{L_1}(\infty) = \frac{1}{2} L_1 i_1^2(\infty) = \frac{1}{2}\, 5\,(1.6)^2 = 6.4\,J$$

$$w_{L_2}(\infty) = \frac{1}{2} L_2 i_2^2(\infty) = \frac{1}{2}\, 20\,(-1.6)^2 = 25.6\,J$$

ENERGY TRAPPED IN INDUCTORS

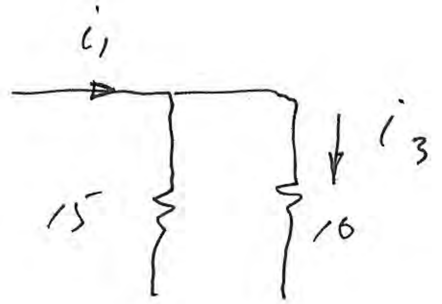
EX 7.2 P219





$$i_0 = \frac{v(t)}{8}$$

$$i_1 = i_0 \frac{40}{40+10}$$ CURRENT DIVISION



$$i_3 = i_1 \frac{15}{15+10}$$

CURRENT DIVISION

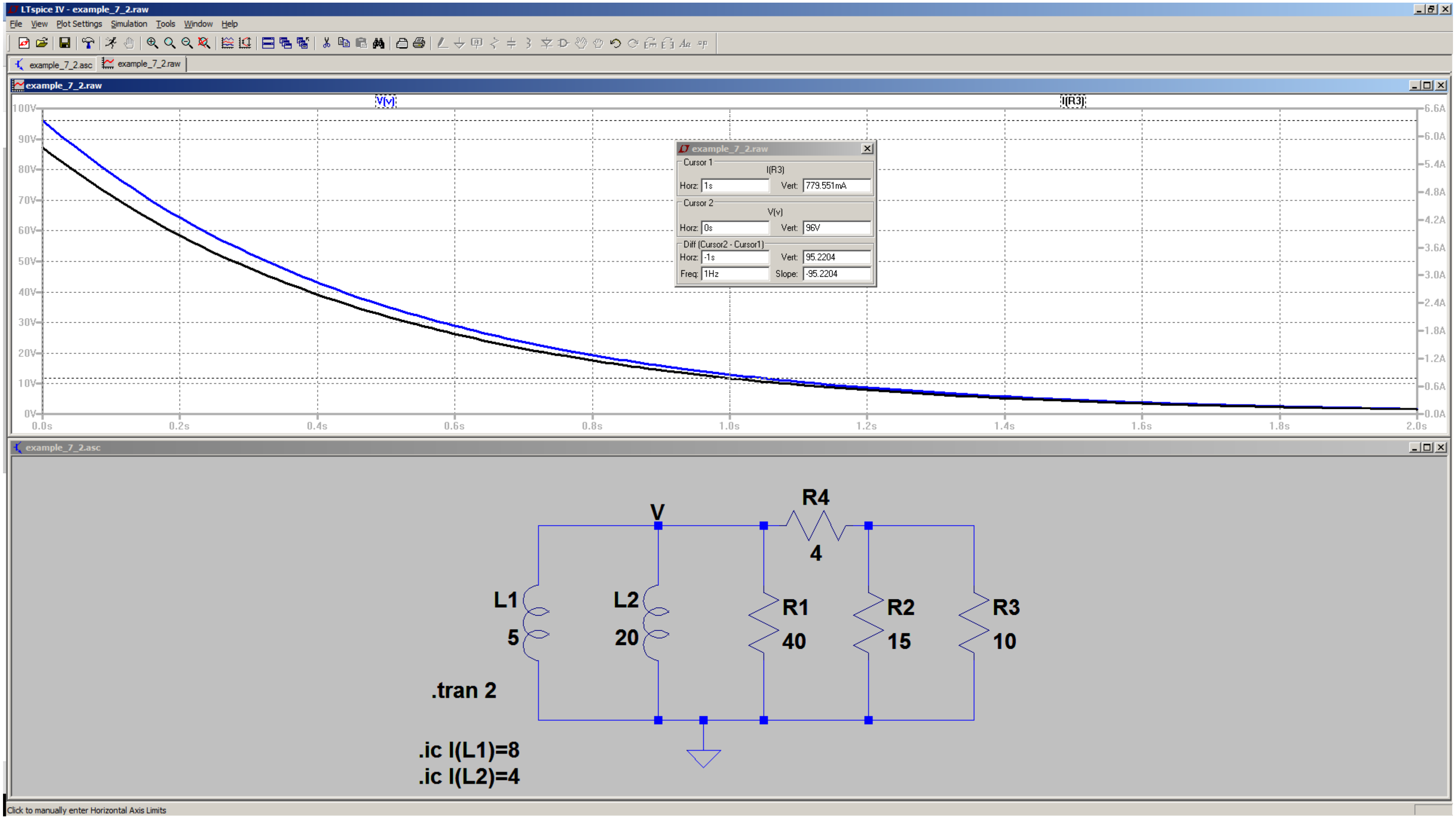
$$i_3 = \frac{15}{15+10} \; \frac{40}{40+10} \; \frac{v(t)}{8} = \frac{v(t)}{10} \; \frac{15}{25}$$

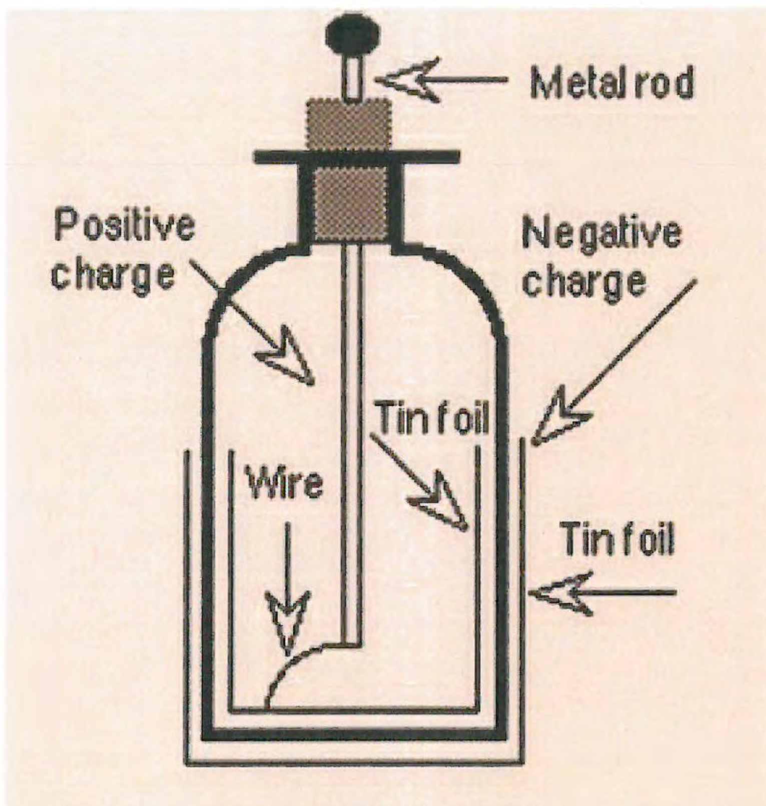
$$i_3(t) = \frac{15}{250} 96 e^{-2t} \, A$$

$$= 5.760 e^{-2t} \, A$$

$$i_3(t)\Big|_{t=1s} = 5.760 \, e^{-2} \, A$$

$$= \underline{\underline{779.5 \, mA}}$$


LTspice IV - example_7_2.raw
File View Plot Settings Simulation Tools Window Help
example_7_2.asc
example_7_2.raw
V(v)
I(R3)
100V
90V
80V
70V
60V
50V
40V
30V
20V
10V
0V
6.6A
6.0A
5.4A
4.8A
4.2A
3.6A
3.0A
2.4A
1.8A
1.2A
0.6A
0.0A
0.0s
0.2s
0.4s
0.6s
0.8s
1.0s
1.2s
1.4s
1.6s
1.8s
2.0s
Cursor 1
I(R3)
Horz: 1s
Vert: 779.551mA
Cursor 2
V(v)
Horz: 0s
Vert: 96V
Diff (Cursor2 - Cursor1)
Horz: -1s
Vert: 95.2204
Freq: 1Hz
Slope: -95.2204
V
R4
4
L1
5
L2
20
R1
40
R2
15
R3
10
.tran 2
.ic I(L1)=8
.ic I(L2)=4
Click to manually enter Horizontal Axis Limits

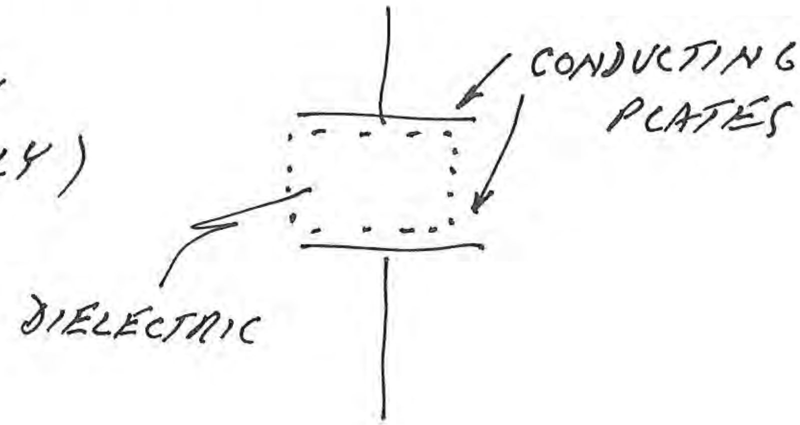


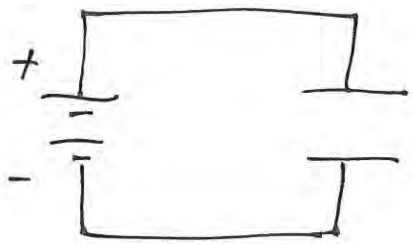
Metal rod
Positive charge
Negative charge
Tin foil
Wire
Tin foil

Leyden jar

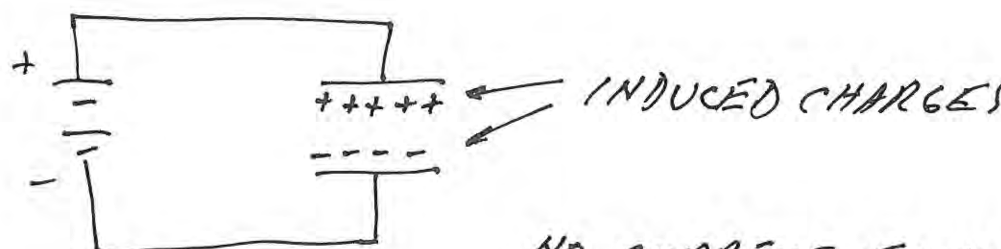
CAPACITOR

PHYSICALLY
(SYMBOLICALLY)





WHAT HAPPENS HERE?



NO CURRENT FLOWS

CAP IS AN OPEN CKT TO A DC SOURCE

v-i RELATIONSHIP

$$i = C\frac{dv}{dt} \Rightarrow \text{IF } v = \text{CONST}, \quad i = 0$$

$$v = Ri$$

$$v = L\frac{di}{dt}$$

$$\frac{dv}{dt} = \frac{1}{C}i$$

HOW DO YOU THINK CAPACITORS COMBINE IN SERIES? IN PARALLEL?

$$i = C\frac{dv}{dt}$$

$$\frac{1}{C}i(t)dt = dv(t)$$

$$\frac{1}{C}i(\tau)d\tau = dv(\tau)$$

$$\frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau = \int_{t_0}^{t} dv(\tau)$$

$$v(t) - v(t_0) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$$

IF $t_0 = 0$

$$v(t) = \frac{1}{C}\int_{0}^{t} i(\tau)d\tau + v(0)$$

$\Rightarrow$ VOLTAGE CANNOT CHANGE INSTANTANEOUSLY

POWER: $p = v \cdot i$

$$p = Cv\frac{dv}{dt}$$

ENERGY $$w(t) = \int_{t_0}^{t} p(\tau)d\tau = C\int_{t_0}^{t} v(\tau)dv(\tau)$$

$$= \frac{1}{2}Cv^2(\tau)\Big|_{t_0}^{t}$$

IF AT t_0, $w(t_0) = 0$, THEN

$$w(t) = \frac{1}{2}Cv^2(t)$$

PHYSICAL PROPERTY OF CAPACITOR IS

"CAPACITANCE" MEASURED IN

FARADS

FARAD = C/V

$$i = C\frac{dv}{dt}$$

AMP

(C/S)

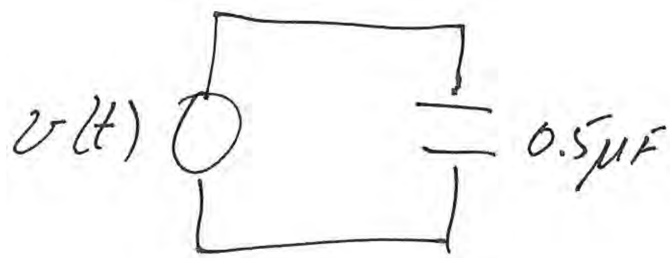
FARAD

(C/V)

VOLTAGE CHANGE

(V/S)

EX 6.4



$$v(t) = \begin{cases} 0 \; ; & t \le 0 \\ 4t\,V \; ; & 0 \le t \le 1s \\ 4e^{-(t-1)}\,V \; ; & t \ge 1s \end{cases}$$

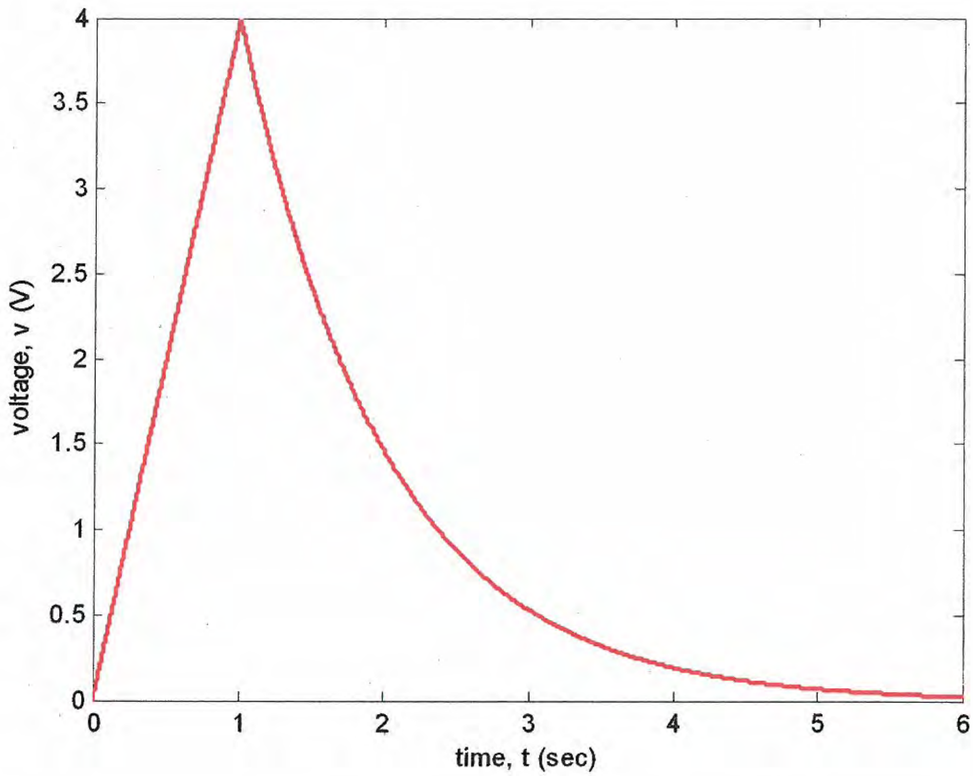
WANT CAP CURRENT, POWER, ENERGY

$$i = C\,\frac{dv}{dt}$$

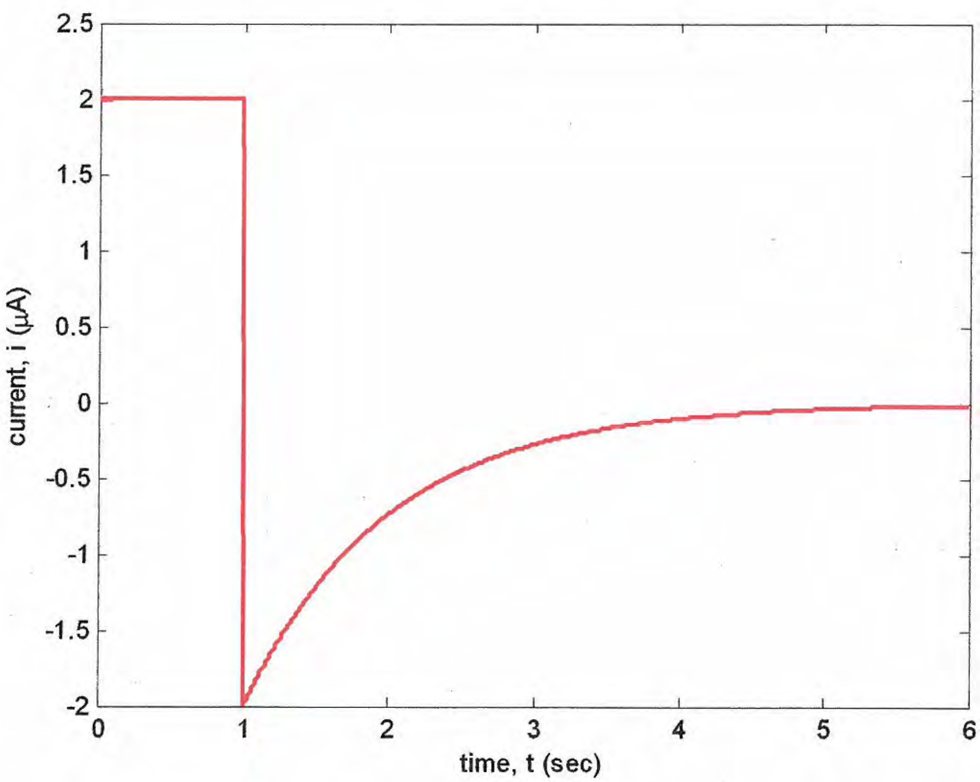
$$i = \begin{cases} (0.5\times10^{-6})\,0 \; ; & t \le 0 \\ (0.5\times10^{-6})\,4 = 2\mu A \; ; & 0 \le t \le 1s \\ (0.5\times10^{-6})(-4e^{-(t-1)}) & \\ \quad = -2e^{-(t-1)}\,\mu A \; ; & t \ge 1s \end{cases}$$

$$
p = \begin{cases} 0 & ; \quad t \le 0 \\ (4t\,V)(2\,\mu A) = 8t\,\mu W & ; \quad 0 \le t \le 1\,s \\ (4e^{-(t-1)}\,V)(-2e^{-(t-1)}\,\mu A) & \\ \quad = -8e^{-2(t-1)}\,\mu W & ; \quad t \ge 1\,s \end{cases}
$$

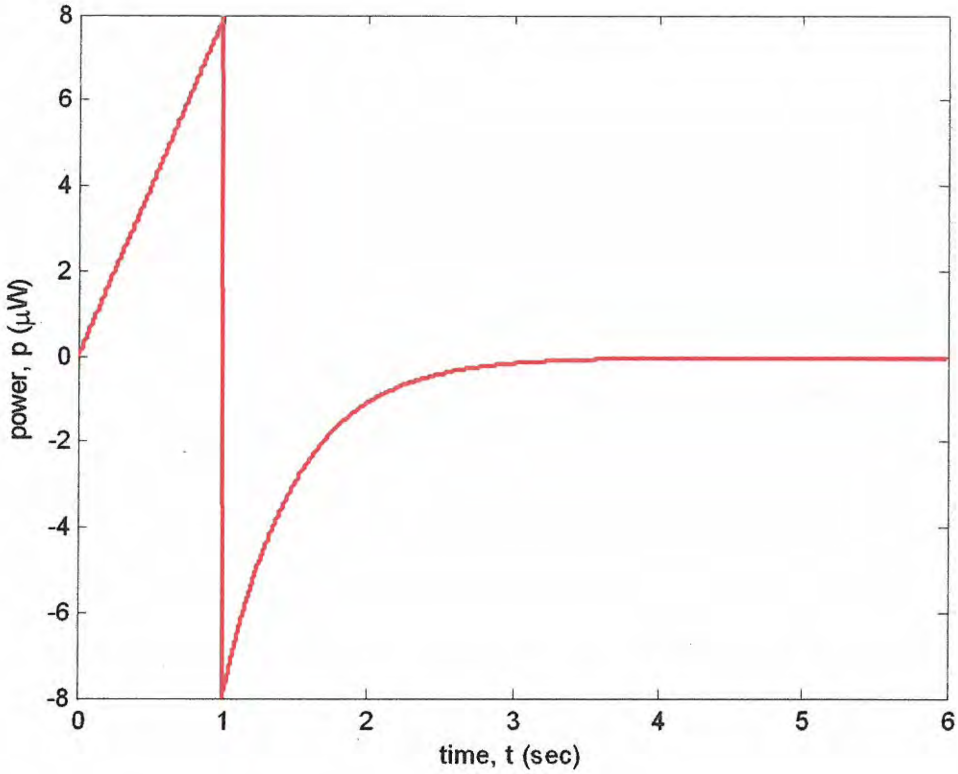
$$
w = \begin{cases} 0 & ; \quad t \le 0 \\ 4t^2\,\mu J & ; \quad 0 \le t \le 1\,s \\ 4e^{-2(t-1)}\,\mu J & ; \quad t \ge 1\,s \end{cases}
$$

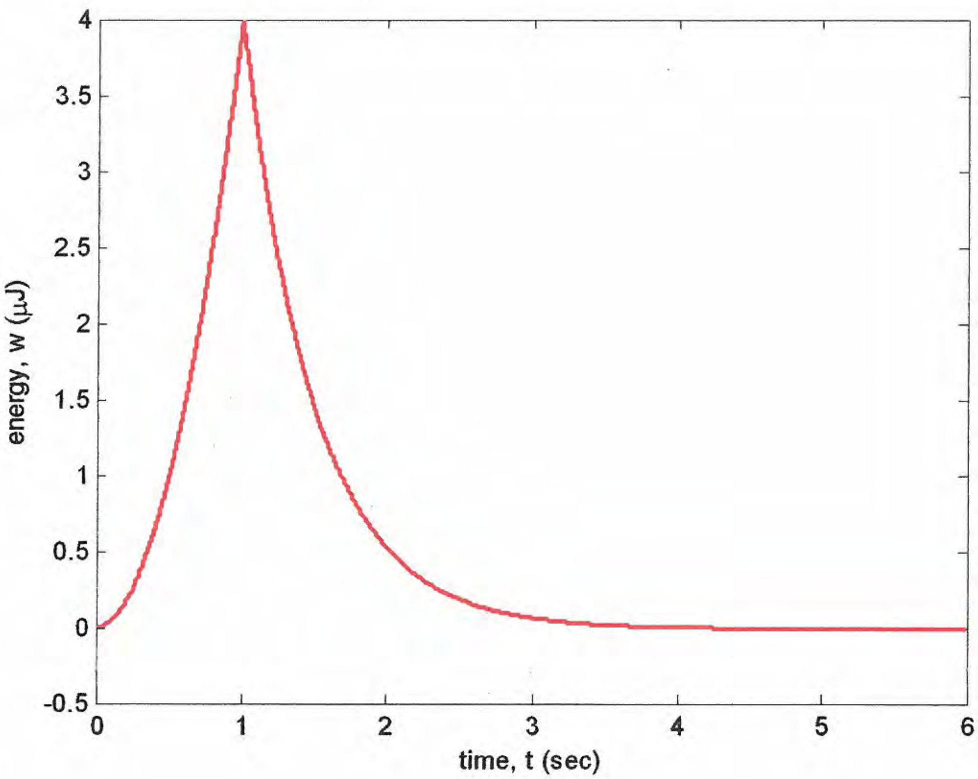


voltage, v (V)
time, t (sec)



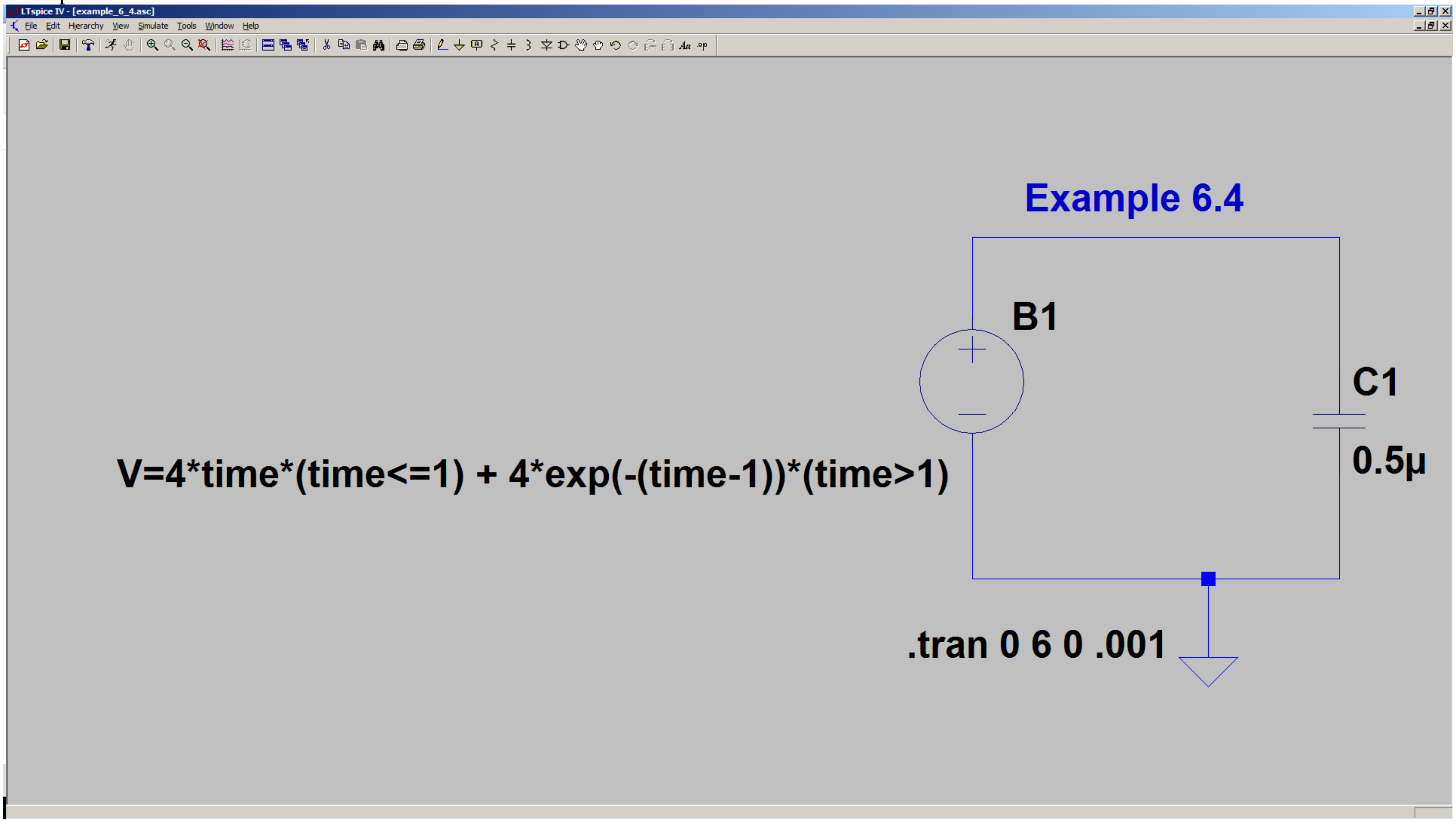
current, i (μA)
time, t (sec)


power, p (μW)
8
6
4
2
0
-2
-4
-6
-8
0
1
2
3
4
5
6
time, t (sec)

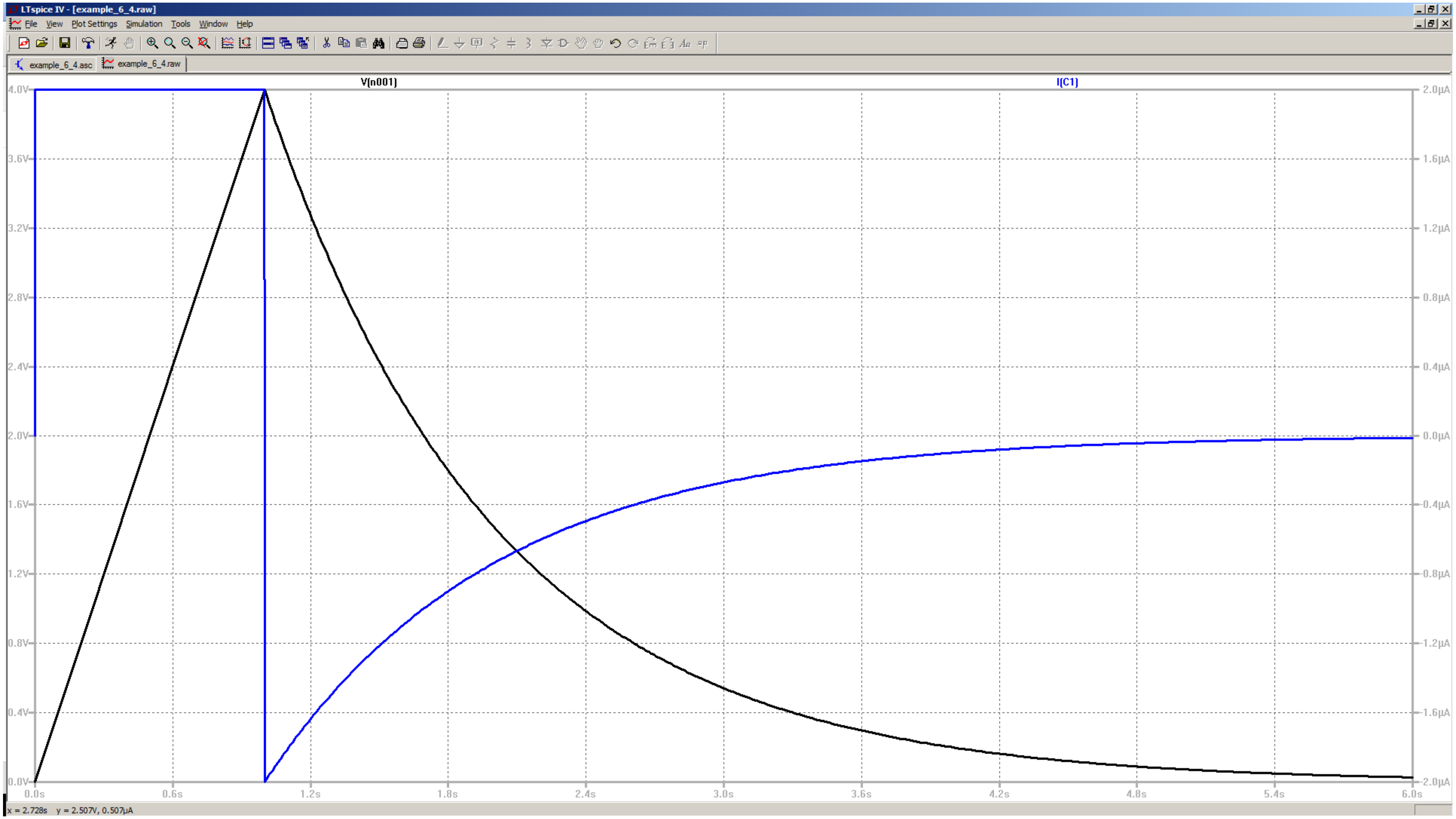

energy, w (μJ)
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
1
2
3
4
5
6
time, t (sec)

```
% example_6_4
% Nilsson & Riedel 10th ed.
% 01/05/15 D D Duncan
%
C = 0.5;% units of micro Farads
dt = 0.001;
t = 0:dt:6.0;% units of sec
v = 4*t .* (0 <= t & t <= 1) + 4*exp(-(t-1)).*(t > 1);% units of V
figure(1);plot(t,v,'r-');
xlabel('time, t (sec)');ylabel('voltage, v (V)');
i = C*diff(v)/dt;% units of microamperes
% shorten v and t by one element
t(end) = [];
v(end) = [];
figure(2);plot(t,i,'r-');
xlabel('time, t (sec)');ylabel('current, i (\muA)');
p = v.*i;
figure(3);plot(t,p,'r-');
xlabel('time, t (sec)');ylabel('power, p (\muW)');
w = cumsum(p)*dt;
figure(4);plot(t,w,'r-');
xlabel('time, t (sec)');ylabel('energy, w (\muJ)');
```

Example 6.4

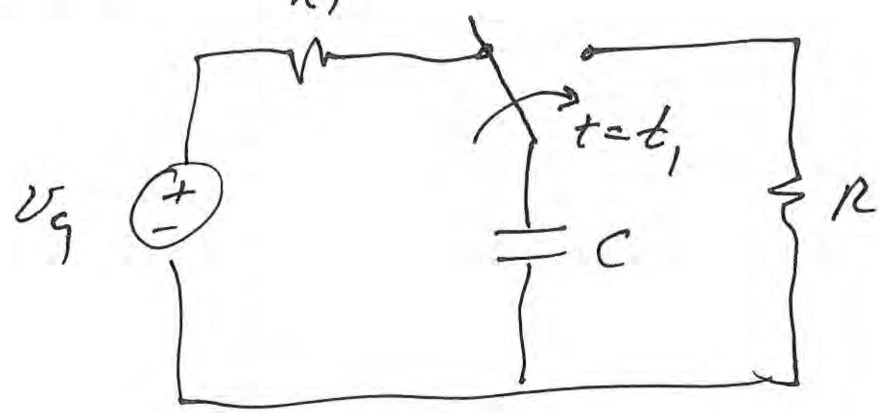


More complex behavioral voltage source

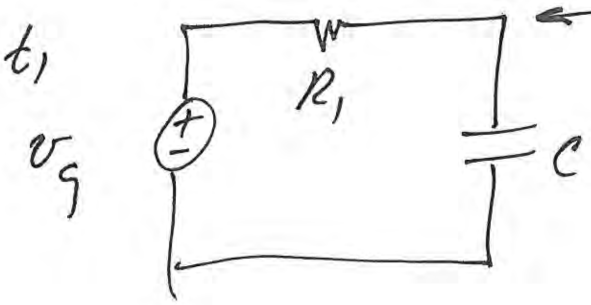


LTspice IV - [example_6_4.raw]
File View Plot Settings Simulation Tools Window Help
example_6_4.asc
example_6_4.raw
V(n001)
I(C1)
4.0V
3.6V
3.2V
2.8V
2.4V
2.0V
1.6V
1.2V
0.8V
0.4V
0.0V
2.0µA
1.6µA
1.2µA
0.8µA
0.4µA
0.0µA
-0.4µA
-0.8µA
-1.2µA
-1.6µA
-2.0µA
0.0s
0.6s
1.2s
1.8s
2.4s
3.0s
3.6s
4.2s
4.8s
5.4s
6.0s
x = 2.728s y = 2.507V, 0.507µA

FIGURE 7.10

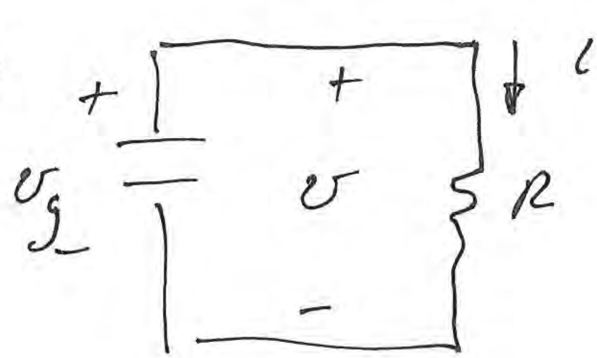


$t < t_1$



VOLTAGE ACROSS CAP IS v_g

$t \geq t_1$



NODAL: $$C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$\frac{C\,dv}{dt} = -\frac{v}{R}$$

$$\int_{t_1}^{t} \frac{dv(x)}{v(x)} = -\int_{t_1}^{t} \frac{dy}{RC}$$

$$\ln v(x) \Big|_{t_1}^{t} = -\frac{(t-t_1)}{RC}$$

$$\ln \frac{v(t)}{v(t_1)} = -\frac{(t-t_1)}{RC}$$

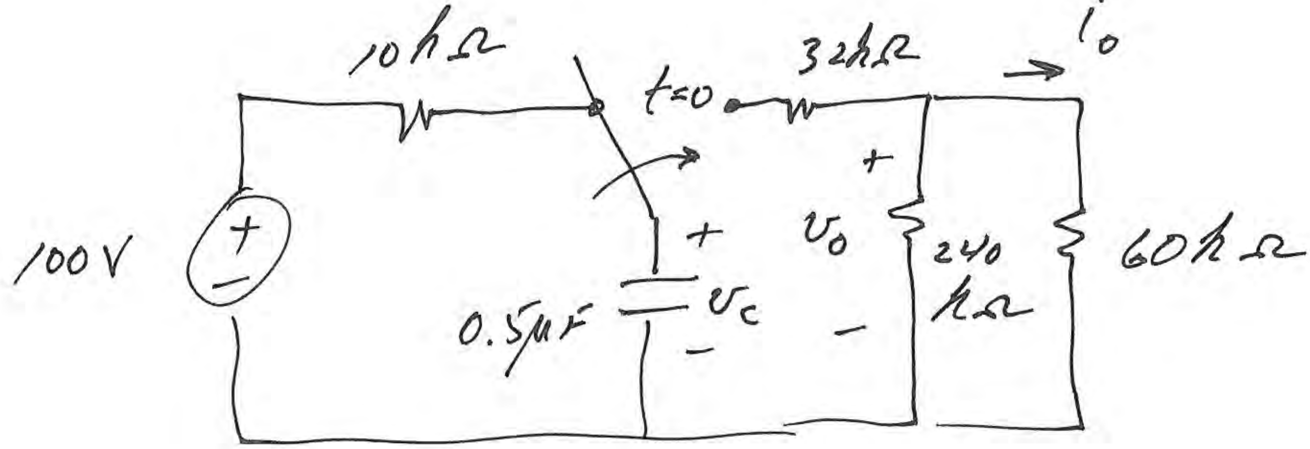
$$v(t) = v(t_1) e^{-(t-t_1)/\tau} \; ; \; t \geq t_1$$

t_1 = SWITCHING TIME

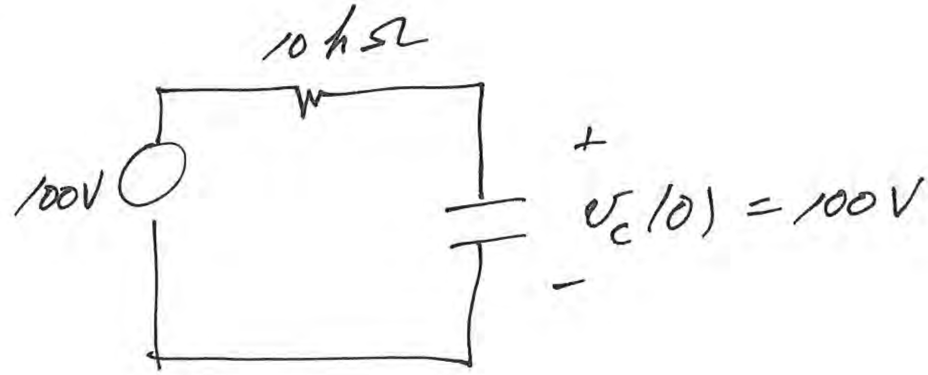
IF $t_1 = 0$

$$v(t) = v(0) e^{-t/\tau} \; ; \; t \geq 0$$

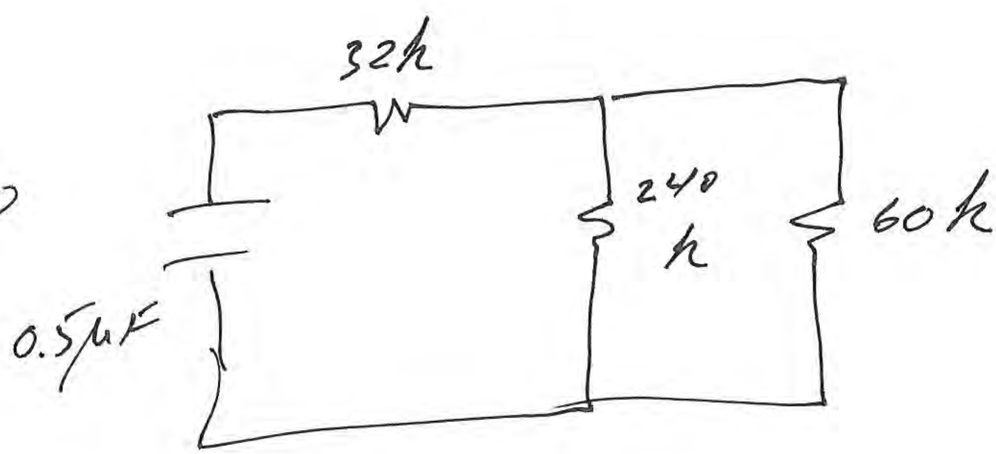
EXAMPLE 7.3

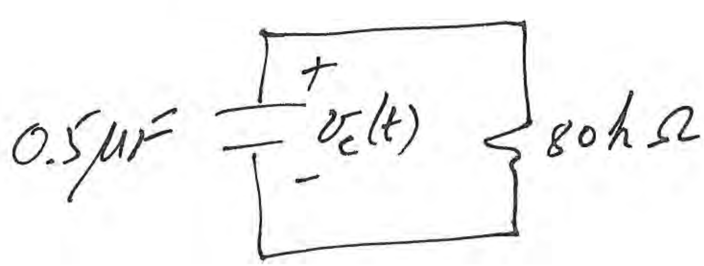


$t < 0$



$t \geq 0$

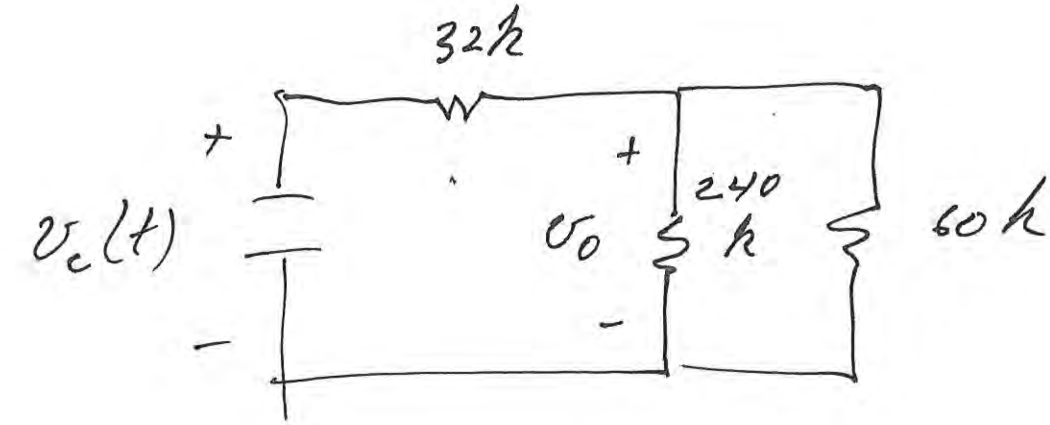


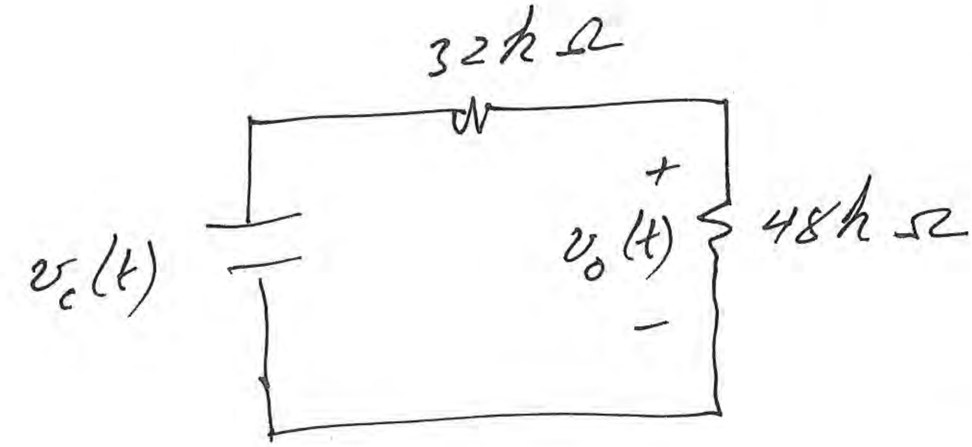


$$\tau = RC = 0.04 \text{ SEC}$$

$$v_c(t) = v_c(0)\, e^{-t/\tau}$$

$$\boxed{v_c(t) = 100\, e^{-25t} \text{ V} \; ; \; t \geq 0}$$

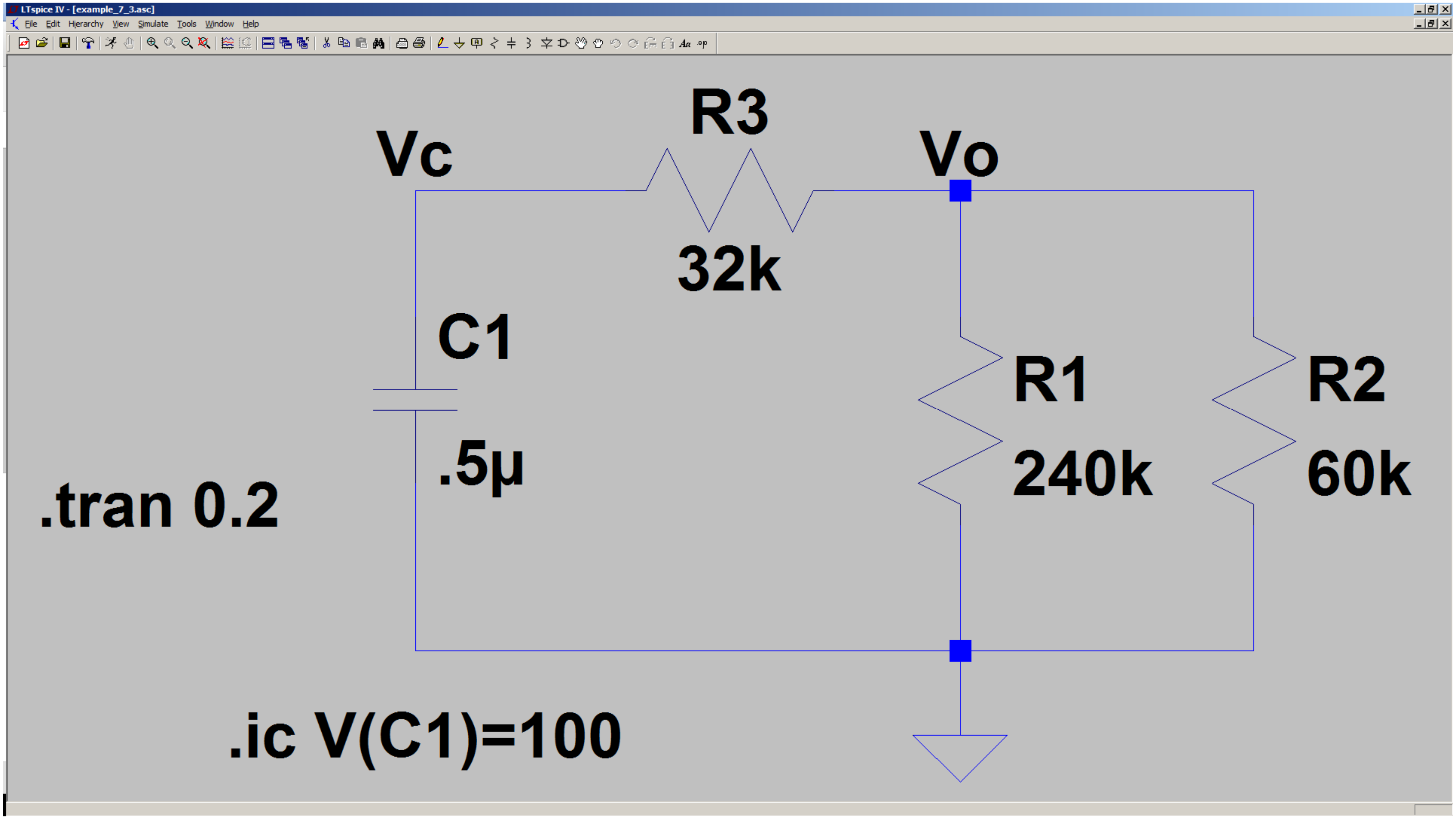


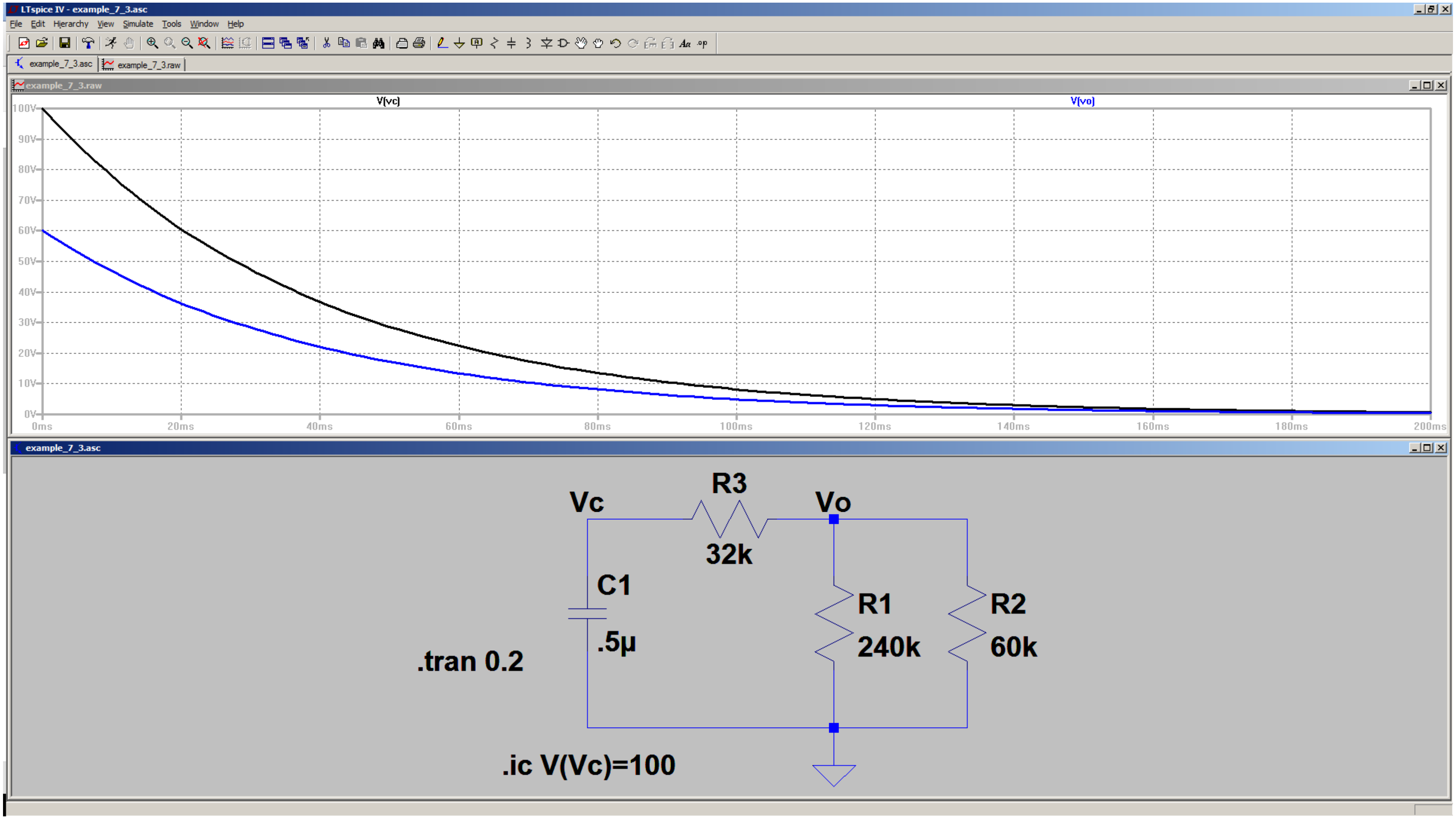


VOLTAGE DIVISION FOR $v_o(t)$:

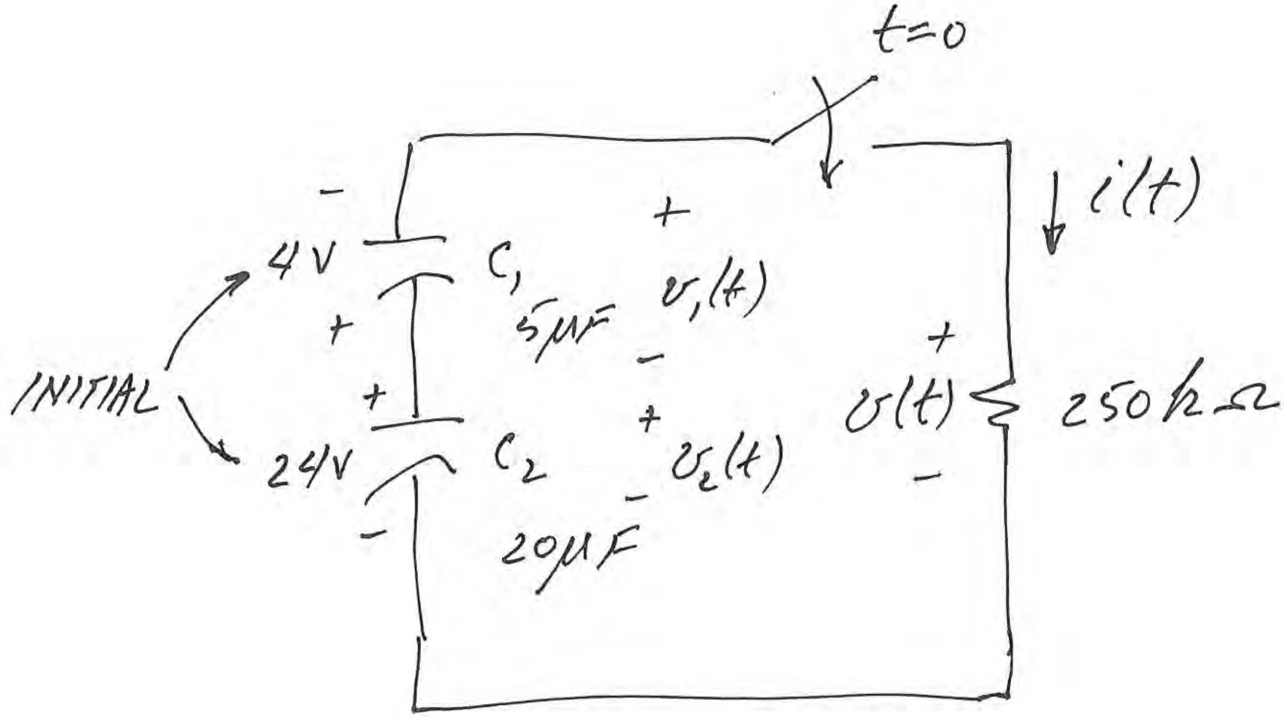
$$v_o(t) = \frac{48k\Omega}{(32k\Omega + 48k\Omega)}\, v_c(t)$$

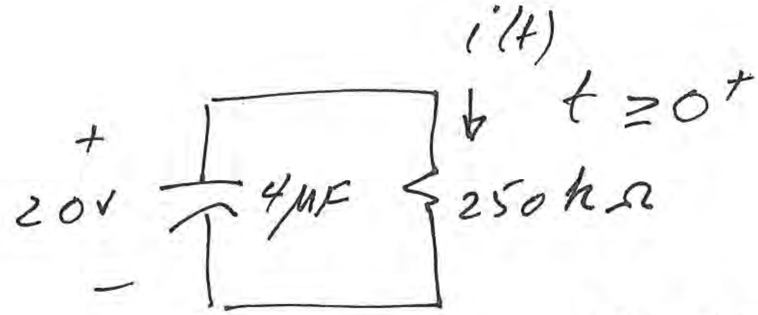
Example 7.3


LTspice IV - [example_7_3.asc]
File Edit Hierarchy View Simulate Tools Window Help
R3
Vc
Vo
32k
C1
.5µ
R1
240k
R2
60k
.tran 0.2
.ic V(C1)=100


LTspice IV - example_7_3.asc
example_7_3.asc
example_7_3.raw
V(vc)
V(vo)
100V
90V
80V
70V
60V
50V
40V
30V
20V
10V
0V
0ms
20ms
40ms
60ms
80ms
100ms
120ms
140ms
160ms
180ms
200ms
Vc
R3
32k
Vo
C1
.5µ
R1
240k
R2
60k
.tran 0.2
.ic V(Vc)=100

EXAMPLE 7.4 PG 223





$\tau = RC = 1S$

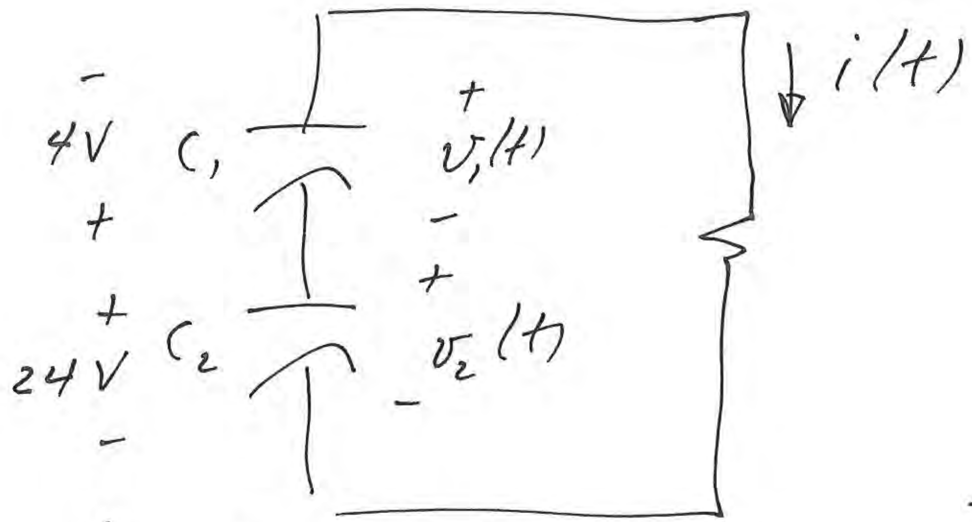
VOLTAGE DIV FOR V_1 & V_2?

$$v(t) = 20e^{-t} \text{ V} ; t \geq 0^+$$

$$i(t) = \frac{v(t)}{25,000} = 80e^{-t} \, \mu A ; t \geq 0^+$$

$$v_1(t) = -\frac{1}{C_1}\int_0^t i(x)dx - 4$$

$$v_2(t) = -\frac{1}{C_2}\int_0^t i(x)dx + 24$$



NOTE SIGN

$$v_1 = -\frac{1}{C_1}\int_0^t i(x)\,dx + \overbrace{v_1(0^+)}^{-4V}$$

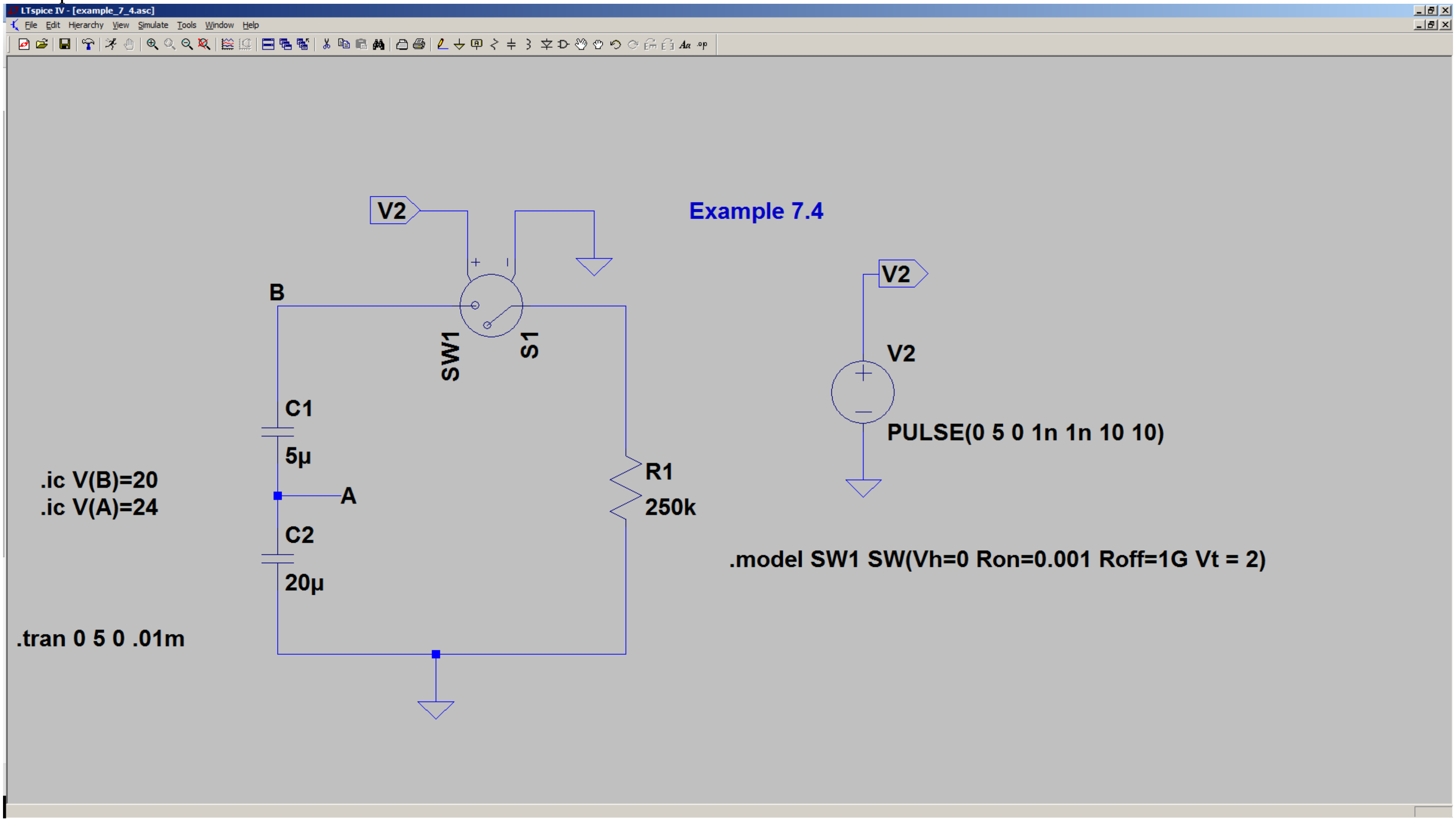
$$v_2 = -\frac{1}{C_2}\int_0^t i(x)\,dx + \overbrace{v_2(0^+)}^{+24V}$$

NOTE SIGN

$$v_1(t) = (16e^{-t} - 20)\ V; \quad t \geq 0$$

$$v_2(t) = (4e^{-t} + 20)\ V; \quad t \geq 0$$

Example 7.4


LTspice IV - [example_7_4.asc]
File Edit Hierarchy View Simulate Tools Window Help
Example 7.4
V2
B
SW1
S1
C1
5µ
A
C2
20µ
R1
250k
V2
V2
PULSE(0 5 0 1n 1n 10 10)
.ic V(B)=20
.ic V(A)=24
.tran 0 5 0 .01m
.model SW1 SW(Vh=0 Ron=0.001 Roff=1G Vt = 2)

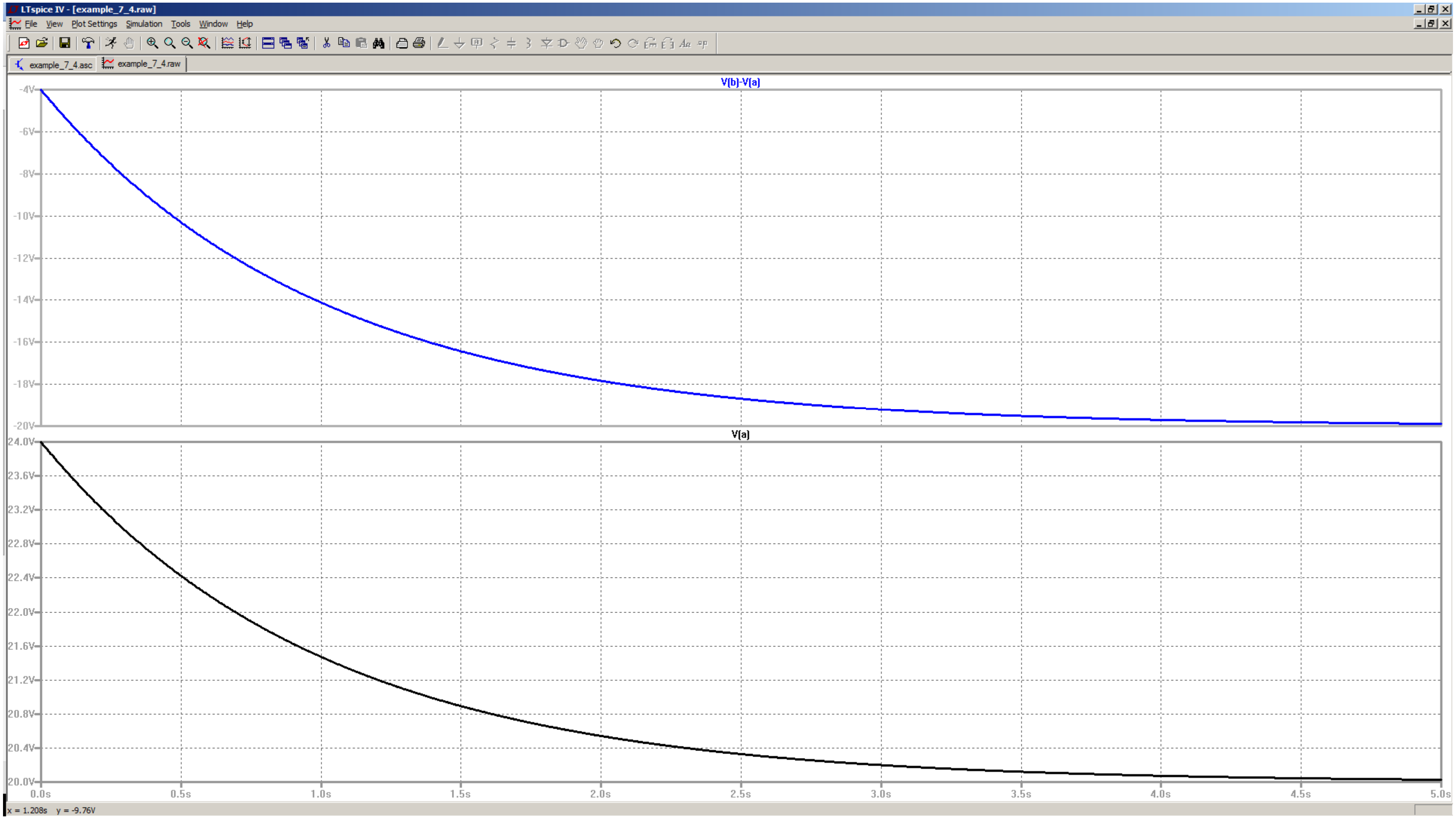
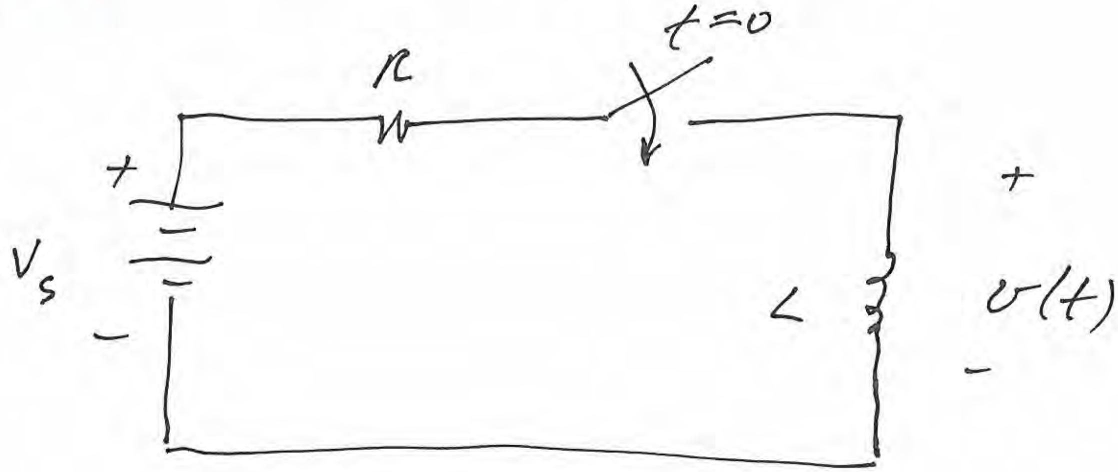

LTspice IV - [example_7_4.raw]
File View Plot Settings Simulation Tools Window Help
example_7_4.asc
example_7_4.raw
V(b)-V(a)
-4V
-6V
-8V
-10V
-12V
-14V
-16V
-18V
-20V
V(a)
24.0V
23.6V
23.2V
22.8V
22.4V
22.0V
21.6V
21.2V
20.8V
20.4V
20.0V
0.0s
0.5s
1.0s
1.5s
2.0s
2.5s
3.0s
3.5s
4.0s
4.5s
5.0s
x = 1.208s y = -9.76V

FIG 7.16 STEP RESPONSE



MESH: $V_s = Ri + L\dfrac{di}{dt}$

$$\frac{di}{dt} = \frac{-Ri + V_s}{L} = -\frac{R}{L}\left(i - V_s/R\right)$$

$$\frac{di(t)}{i(t) - V_s/R} = -\frac{dt}{\tau} \qquad \tau = L/R$$

$$\frac{di(x)}{i(x) - V_s/R} = -\frac{dx}{\tau}$$

$$\int_{t_0}^{t} \frac{di(x)}{i(x) - V_s/R} = -\frac{1}{\tau}\int_{t_0}^{t} dx$$

$$\ln\left[i(x) - V_s/R\right]\Big|_{t_0}^{t} = -\frac{1}{\tau}(t - t_0)$$

$$\ln\left[\frac{i(t) - V_s/R}{i(t_0) - V_s/R}\right] = -\frac{1}{\tau}(t - t_0)$$

$$i(t) = \frac{V_s}{R} + \left[i(t_0) - \frac{V_s}{R}\right] e^{-(t-t_0)/\tau}$$

THIS PROBLEM, $t_0 = 0$

$$i(t) = \frac{V_s}{R} + \left[i(0) - \frac{V_s}{R}\right] e^{-t/\tau}$$

NOTE: $i(0) = \frac{V_s}{R} + \left[i(0) - \frac{V_s}{R}\right] = i(0)$

$i(\infty) = V_s/R$

SOLUTION IS OF FORM

$$i(t) = i(\infty) + \left[i(0) - i(\infty)\right] e^{-t/\tau}$$

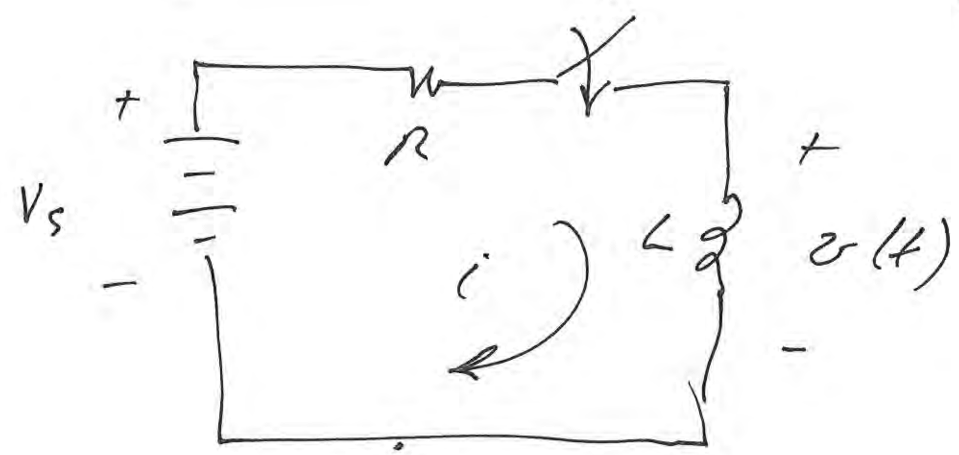
THIS PARTICULAR PROBLEM

$$i(0) = 0$$

$$\therefore \quad i(t) = \frac{V_s}{R}\left(1 - e^{-t/\tau}\right) \qquad \tau = L/R$$

$$t \geq 0$$

FIGURE 7.16



ASSUME NON-ZERO $i(0^-)$

LOOP: $V_S - iR - L\dfrac{di}{dt} = 0$

$$\frac{di}{dt} = \frac{V_S - iR}{L} = -\frac{R}{L}\left(i - V_S/R\right)$$

$$\frac{di}{i - V_S/R} = -\frac{dt}{\tau} \quad (\tau = L/R)$$

INTEGRATE: $\ln\left[i - V_S/R\right]\Big|_{i(0^+)}^{i(t)} = -t/\tau$

$\vdots$

$$i(t) = \frac{V_S}{R} + \left[i(0^+) - \frac{V_S}{R}\right]e^{-t/\tau}$$

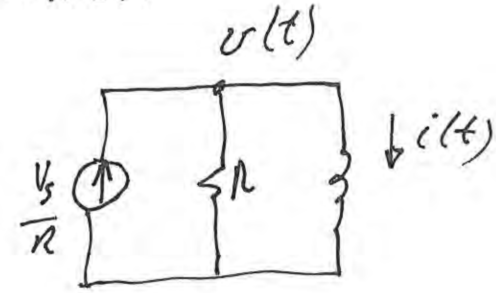
$$v(t) = L \frac{di(t)}{dt}$$

$$v(t) = L \frac{d}{dt} \left\{ \frac{V_S}{R} + \left[i(0^+) - \frac{V_S}{R} \right] e^{-t/\tau} \right\}$$

$$= L \left[i(0^+) - \frac{V_S}{R} \right] \left(-\frac{1}{\tau} \right) e^{-t/\tau}$$

$$v(t) = \left[V_S - R\, i(0^+) \right] e^{-t/\tau} \qquad \text{(EQ 7.42)}$$

ALTERNATIVE APPROACH



NODAL: $i(t) = \dfrac{V_s - v(t)}{R}$

$\dfrac{d}{dt}$ BOTH SIDES: $\dfrac{di(t)}{dt} = -\dfrac{1}{R}\dfrac{dv(t)}{dt}$

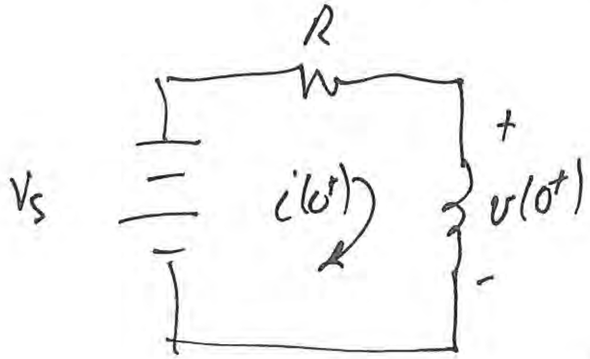
BUT $v(t) = L\dfrac{di}{dt}$

$$\frac{1}{L}v(t) + \frac{1}{R}\frac{dv(t)}{dt} = 0$$

$$\frac{dv(t)}{v(t)} = -\frac{dt}{\tau}$$

INTEGRATE: $\ln\left[\dfrac{v(t)}{v(0^+)}\right] = -t/\tau$

$$v(t) = v(0^+)e^{-t/\tau}$$



$V_s - i(0^+)R - v(0^+) = 0$

$v(t) = [V_s - Ri(0^+)]e^{-t/\tau}$

(SAME AS EQ 7.42)

SUMMARY

NODAL EQ FOR VOLTAGE

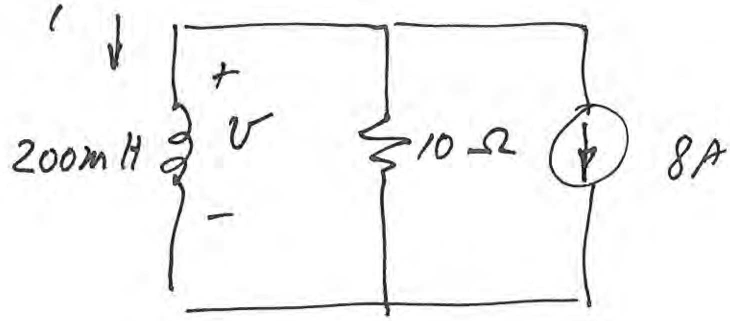
$$\frac{dv(t)}{dt} + \frac{R}{L} v(t) = 0$$

LOOP EQ FOR CURRENT

$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V_S}{L}$$

? DIFFERENCE IN SOLUTIONS

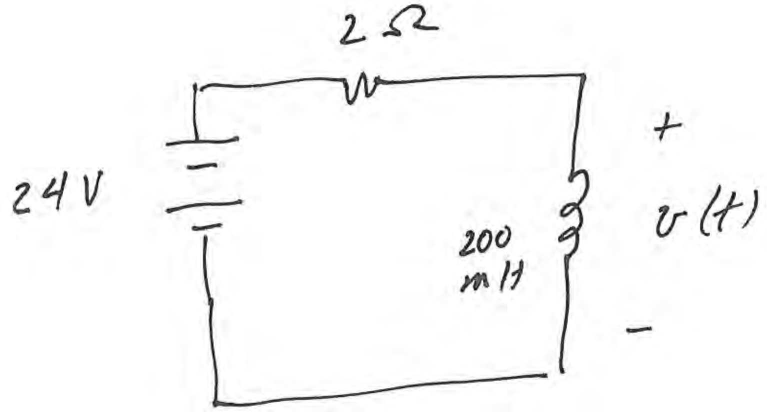
EXAMPLE 7.5



$t < 0$

$i(0^-) = -8A$

$v(0^-) = 0V$



$t \geq 0$

$i(0^+) = -8A$

$v(0^+) = ?$

$$i(\infty) = \frac{24V}{2\Omega} = 12A$$

$$\tau = L/R = 0.2/2 = 0.1S$$

KNOW SOL'N OF FORM

$$i(t) = A + Be^{-t/\tau}$$

$$i(t) = A + Be^{-t/\tau}$$

$$\left.\begin{aligned} i(0^+) &= A + B \\ i(\infty) &= A \end{aligned}\right\} \Rightarrow B = i(0^+) - i(\infty)$$

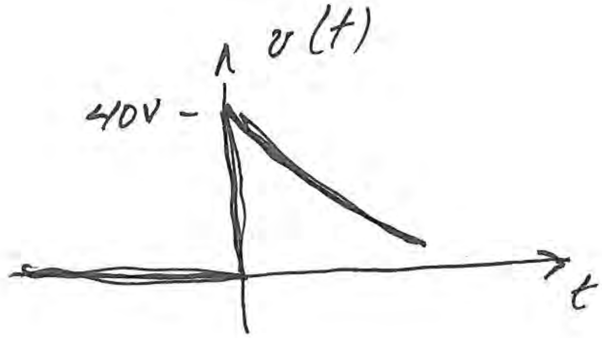
$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$$

$$i(t) = 12 + [-8 - 12]e^{-10t}$$

$$i(t) = 12 - 20e^{-10t}\ A \ ; \ t \geq 0^+$$

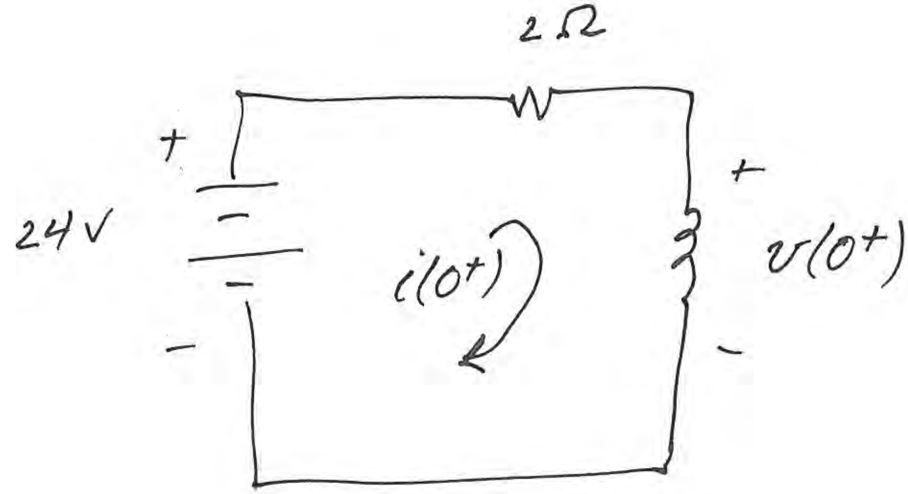
$$v(t) = L\frac{di(t)}{dt} = 0.2H\left(200e^{-10t}\ A/s\right)$$

$$v(t) = 40e^{-10t}\ V \ ; \ t \geq 0^+$$



$$v(0^-) = 0$$

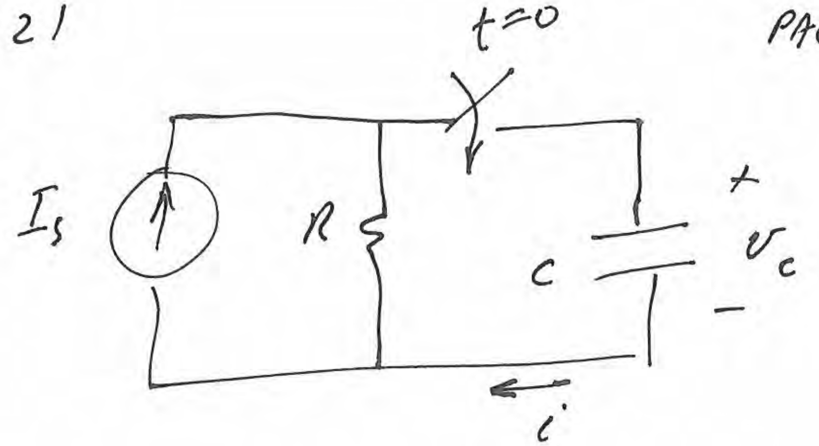
CONSIDER INSTANT AFTER WHICH SWITCH IS THROWN



LOOP EQ $\quad 24V - 2\Omega\, i(0^+) = v(0^+)$

$$24V + 16V = v(0^+)$$

FIGURE 7.21 PAGE 229



NODAL EQ: $C\dfrac{dv_c}{dt} + \dfrac{v_c}{R} - I_s = 0$

$$\frac{dv_c}{dt} + \frac{1}{RC}v_c = \frac{I_s}{C}$$

SOL'N MUST BE

$$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right]e^{-t/\tau}$$

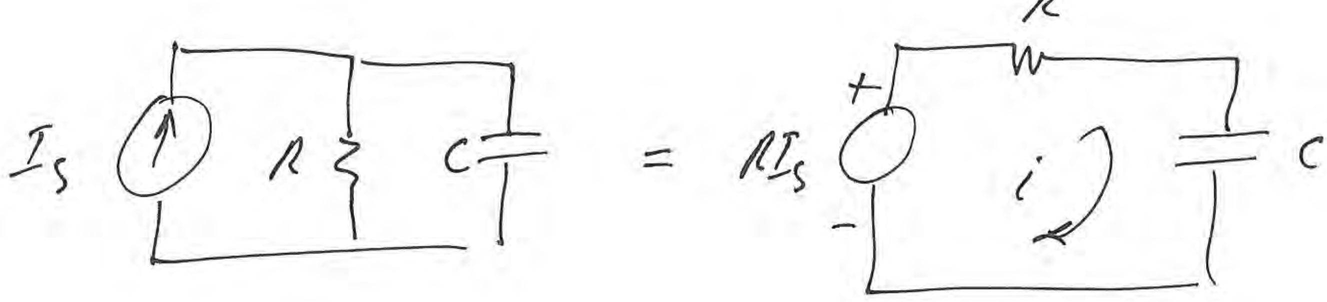
$$v_c(\infty) = RI_s$$

$$v_c(0^+) \equiv V_0 = v_c(0^-)$$

SHOULD BE $t \geq 0$

$$v_c(t) = RI_s + (V_0 - RI_s)e^{-t/RC}\;, \; t \geq 0^+$$

EQUALLY VALID APPROACH



LOOP EQ: $RI_s - Ri - \frac{1}{C}\int i(t)\,dt = 0$

$\frac{d}{dt}$ BOTH SIDES:

$$-R\frac{di}{dt} - \frac{i(t)}{C} = 0$$

$$\frac{di(t)}{dt} = -\frac{1}{RC}\, i(t)$$

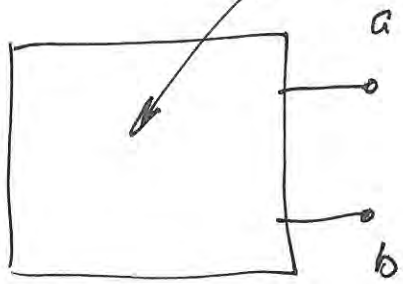
$$\frac{di(t)}{i(t)} = -\frac{dt}{\tau}$$

$\vdots$

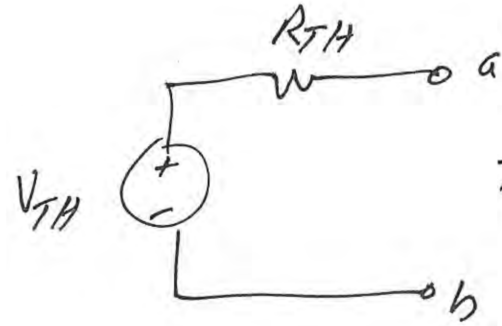
$$i(t) = i(0^+)e^{-t/RC} \quad ; \; t \geq 0^+$$

$I_s - V_0/R$ (FROM MESH EQ
$RI_s - Ri(0^+) - V_0 = 0$)

RESISTIVE NETWORK
w DEP, & INDP SOURCES

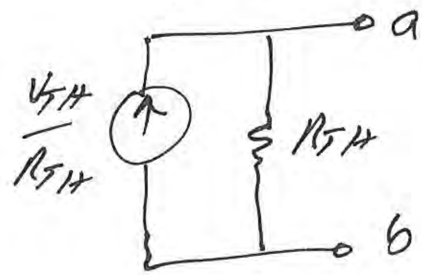


=

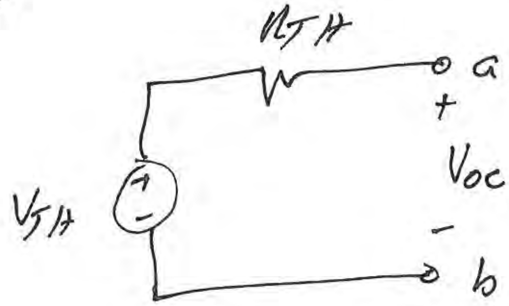


THÉVENIN

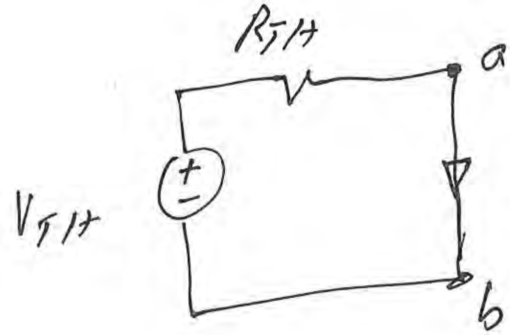
NORTON



CALCULATION :



$$\underline{\underline{V_{TH} = V_{OC}}}$$

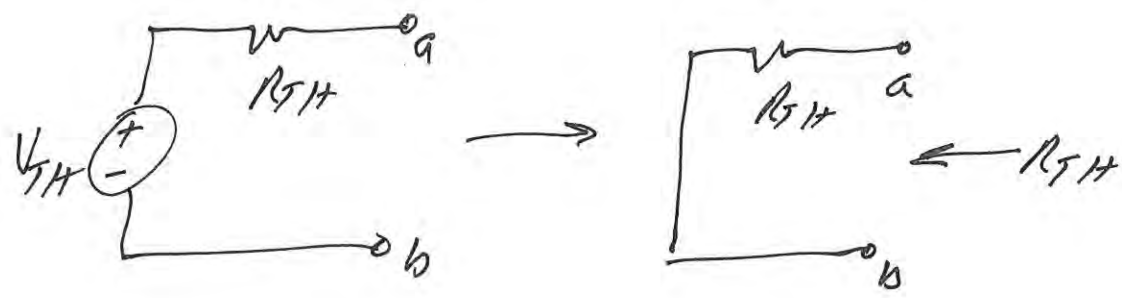


$$\underline{\underline{I_{SC} = \frac{V_{TH}}{R_{TH}}}}$$

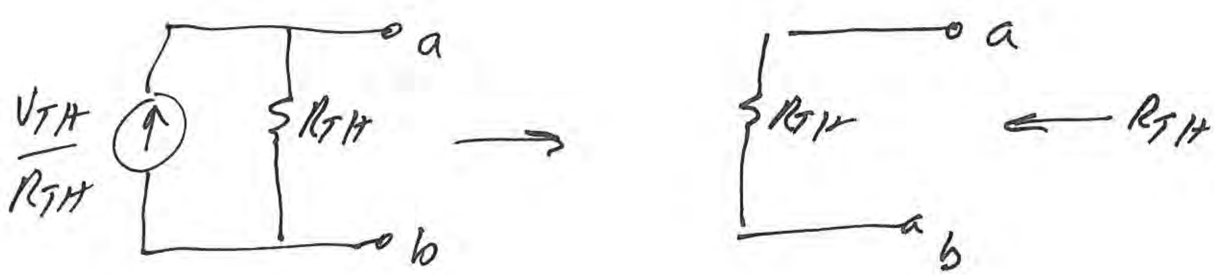
$$\boxed{\begin{aligned} R_{TH} &= \frac{V_{OC}}{I_{SC}} \\ V_{TH} &= V_{OC} \end{aligned}}$$

INDP SOURCES ONLY

SHORT VOLTAGE SOURCES



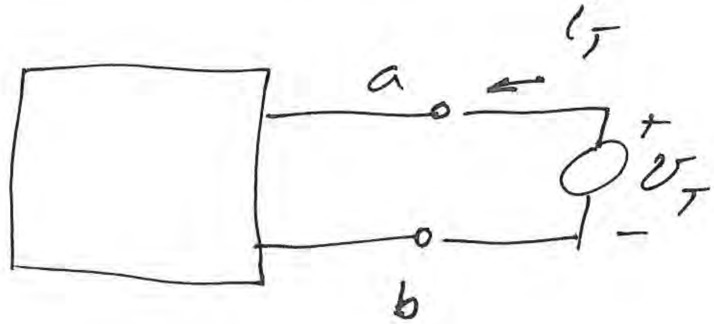
OPEN CURRENT SOURCES



DEP SOURCES

DEACTIVATE ALL <u>INDP</u> SOURCES

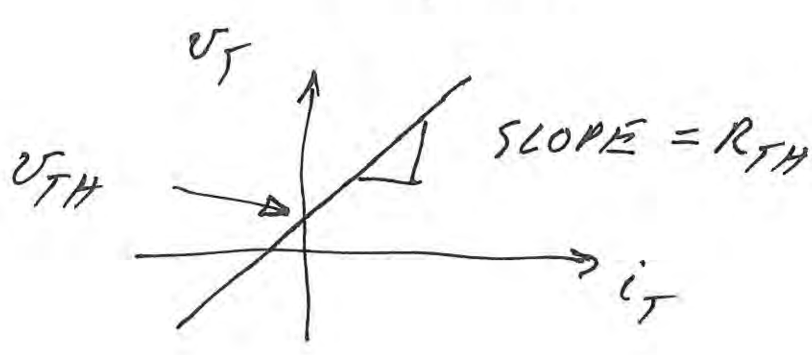
APPL TEST SOURCE

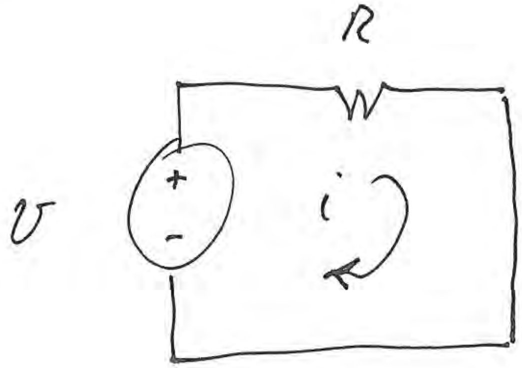


$$R_{TH} = \frac{v_T}{i_T}$$

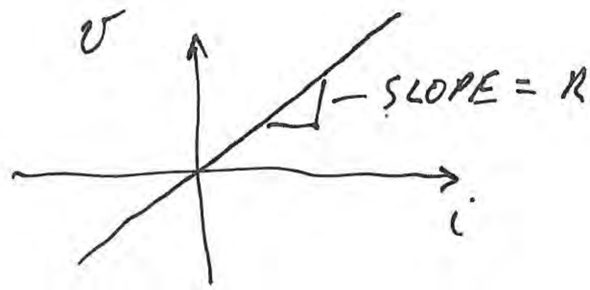
WITH <u>NO</u> DEACTIVATION, GENERALLY GET SOLUTION OF FORM

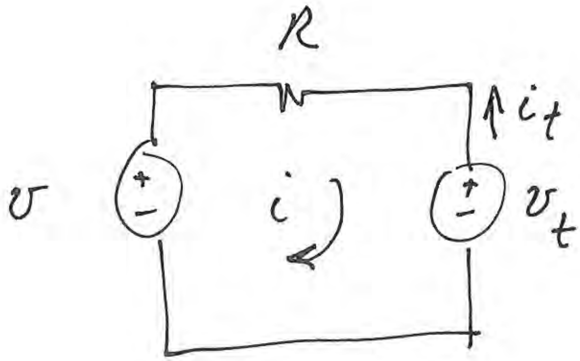
$$v_T = A i_T + B$$





$$v = iR$$



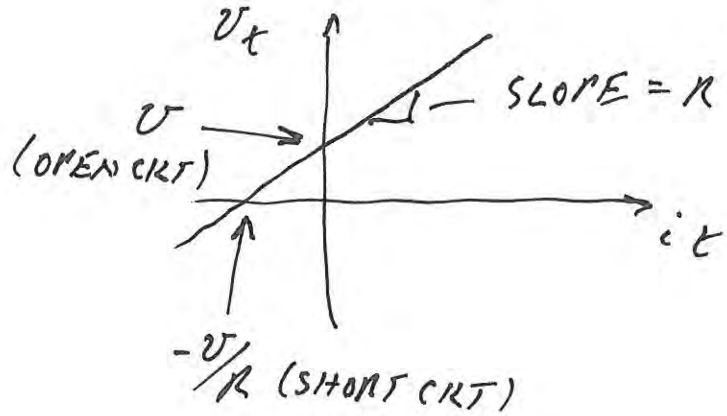


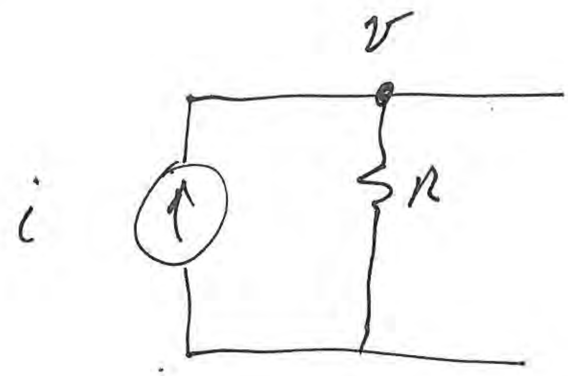
$$v - iR - v_t = 0$$

$$i = -i_t$$

$$v + i_t R - v_t = 0$$

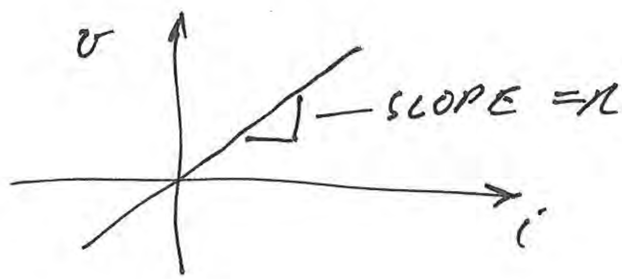
$$v_t = i_t R + v$$

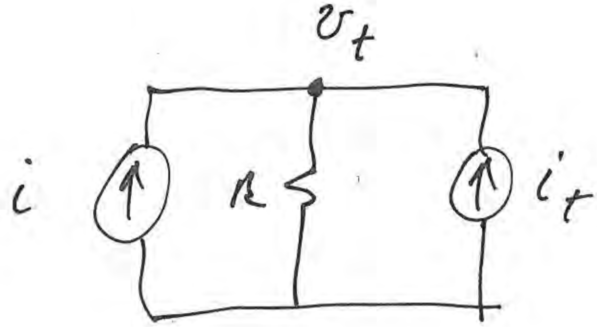




$$\frac{v}{R} - i = 0$$

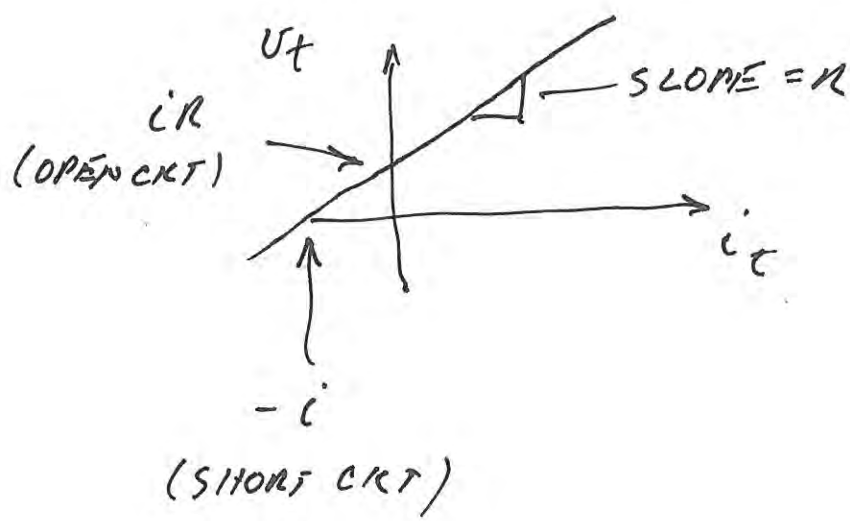
$$v = iR$$



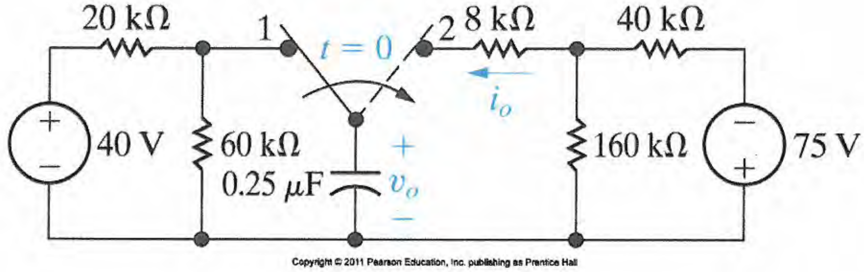


$$\frac{v_t}{R} - i - i_t = 0$$

$$v_t = iR + i_t R$$



EXAMPLE 7.6

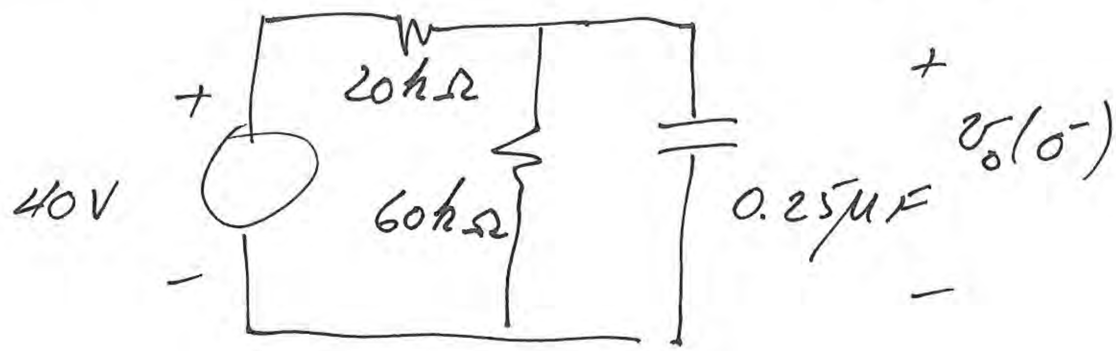


Find

a) $v_o(t)$; $t \geq 0$

b) $i_o(t)$; $t \geq 0^+$

EXAMPLE 7.6 PAGE 230 $t < 0$

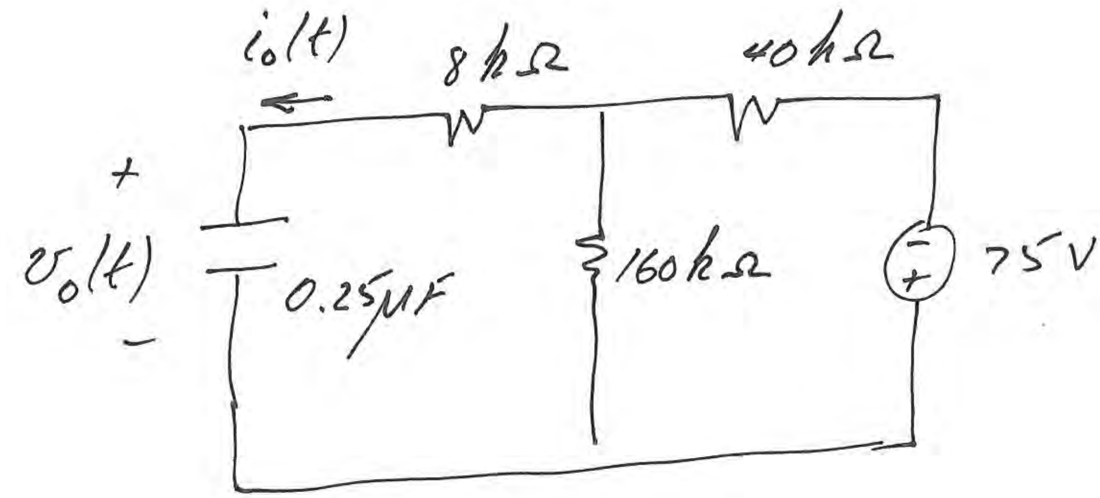


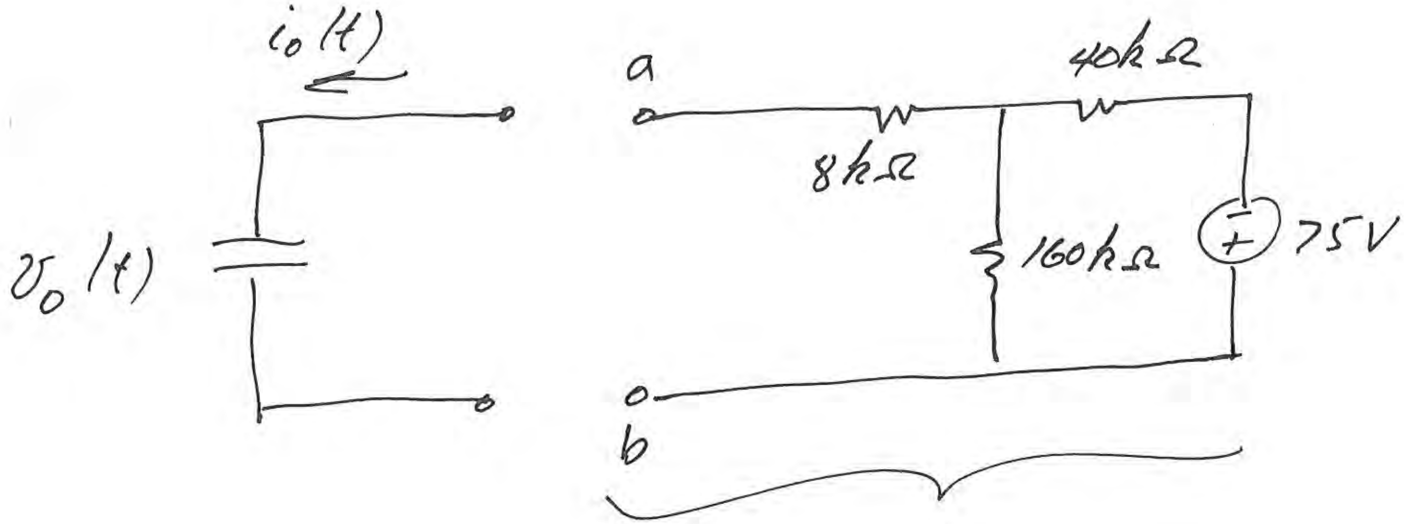
VOLTAGE DIVISION FOR $v_o(0^-)$

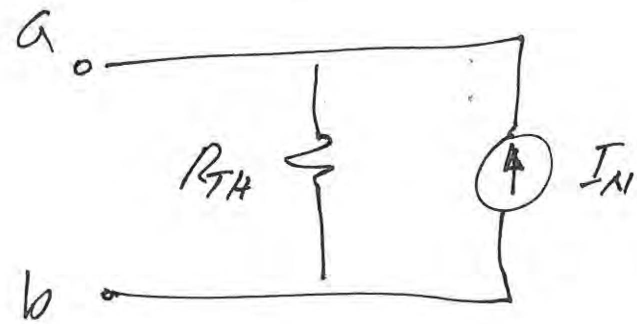
$$v_o(0^-) = 40V\left(\frac{60k\Omega}{20k\Omega + 60k\Omega}\right)$$

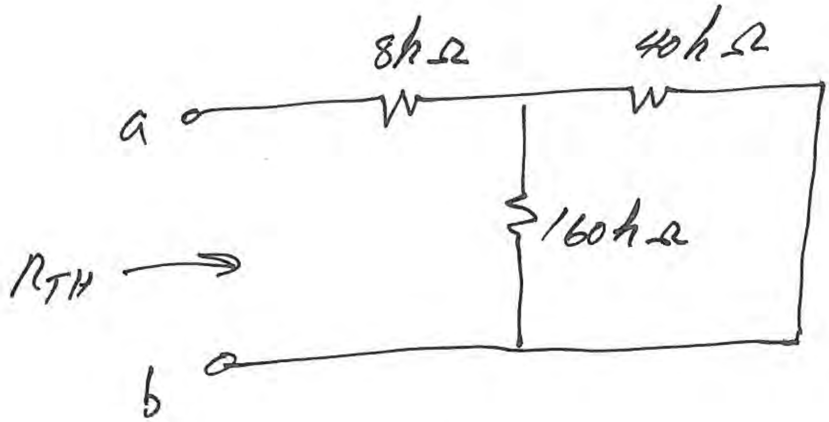
$$v_o(0^-) = 30V = v_o(0^+)$$

$t \geq 0$



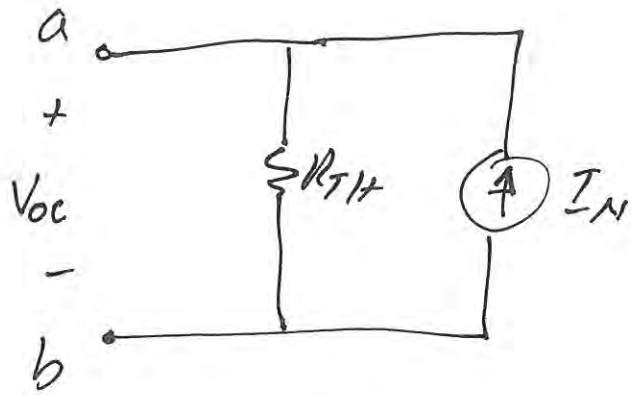






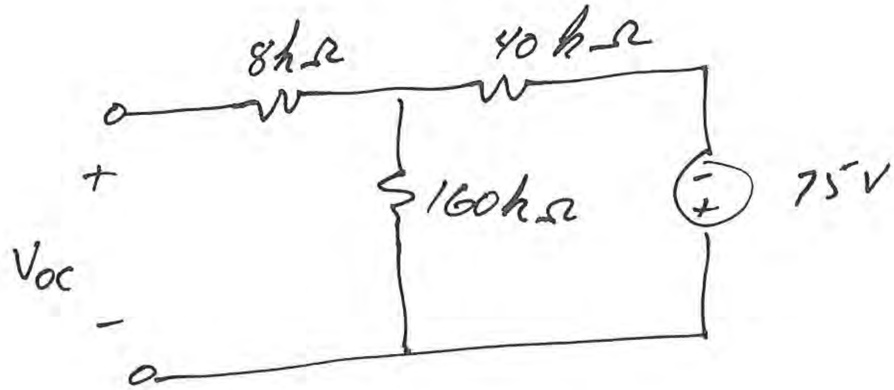
$$R_{TH} = (40k\Omega \parallel 160k\Omega) + 8k\Omega$$

$$= 40k\Omega$$



$$V_{oc} = R_{TH} I_N$$

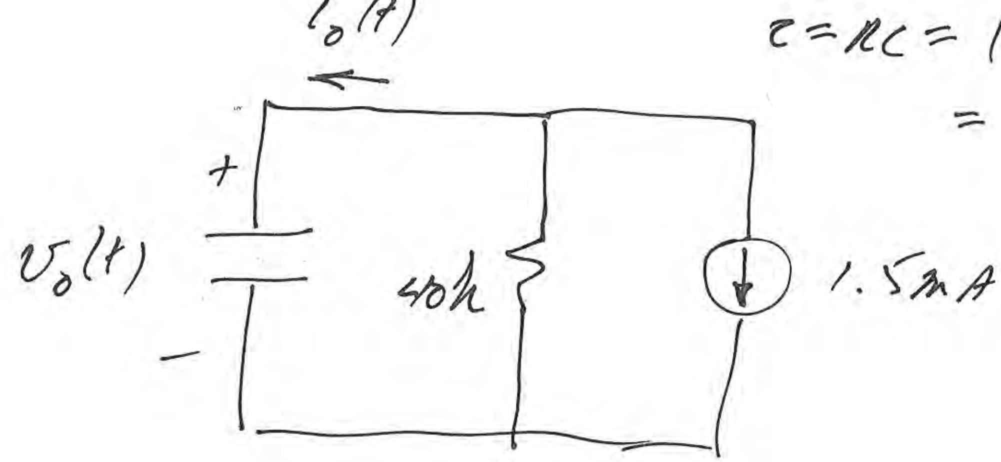
$$I_N = \frac{V_{oc}}{R_{TH}} = ?$$



$$V_{oc} = -75V\left(\frac{160k\Omega}{160k\Omega + 40k\Omega}\right)$$

$$V_{oc} = -60V$$

$$\Rightarrow I_N = \frac{-60V}{40k\Omega} = -1.5mA$$



$$\tau = RC = (40k\Omega)(0.25\mu F)$$
$$= 10ms$$

$$v_o(\infty) = -1.5mA\,(40k\Omega) = -60V$$

$$v_o(t) = v_o(\infty) + \left[v_o(0) - v_o(\infty)\right]e^{-t/\tau}$$

$$= -60 + \left[30 + 60\right]e^{-100t}$$

$$v_o(t) = -60 + 90e^{-100t} \; ; \; t \geq 0$$

$$i_o(t) = C \frac{d}{dt} v_o(t)$$

$$= 0.25\mu F \frac{d}{dt}\left[-60 + 90 e^{-100t}\right]$$

$$= (0.25\mu F)(-9{,}000 e^{-100t} \text{ V/s})$$

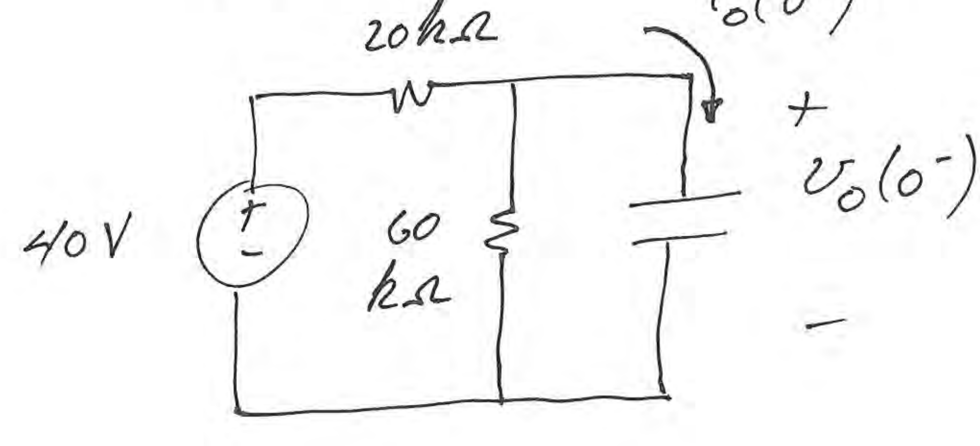
$$i_o(t) = -2.25 e^{-100t} \text{ mA}$$

? RANGE OF VALIDITY ON t

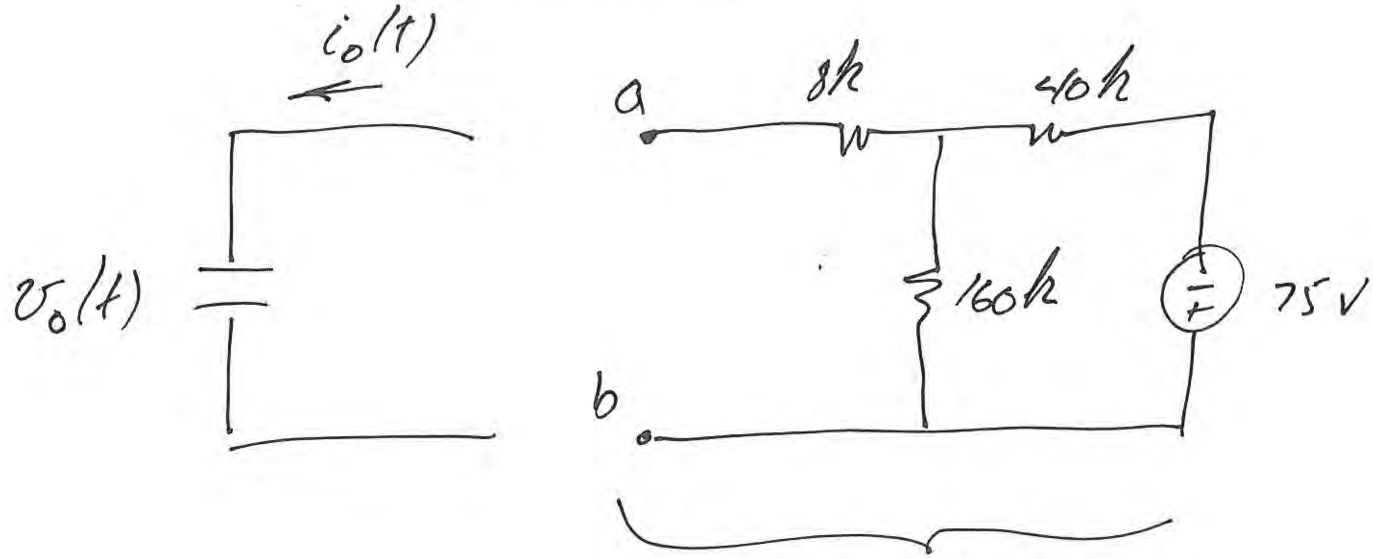
ANS: $t \geq 0^+$

WHY?

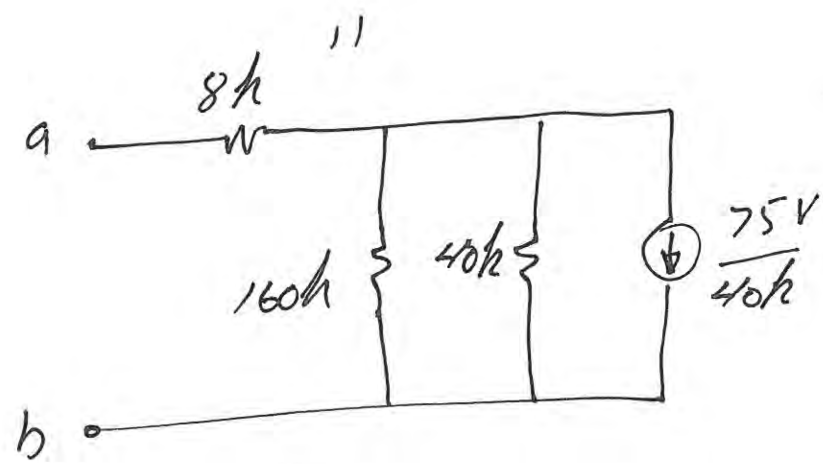
ANS:



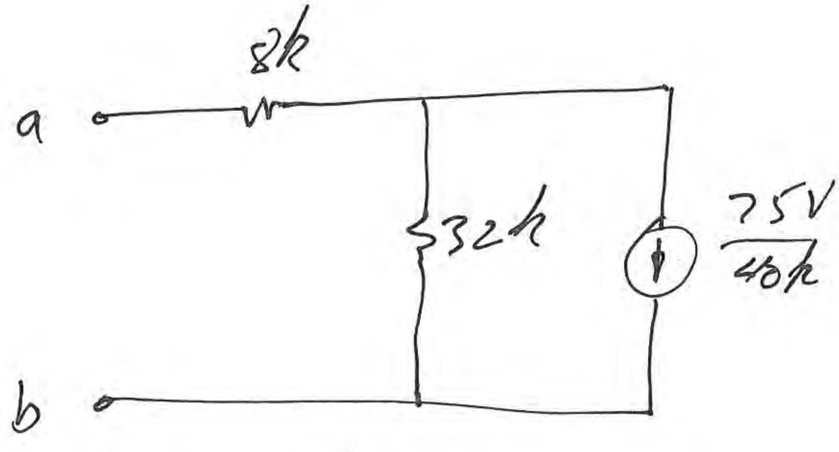
ALTERNATE



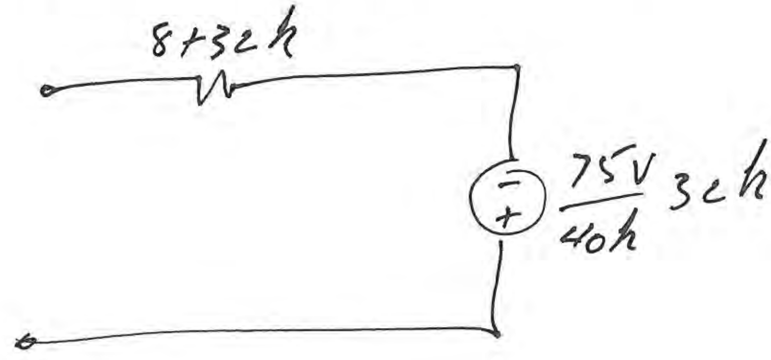
$i_o(t)$
$v_o(t)$
a
b
8k
40k
160k
75 V



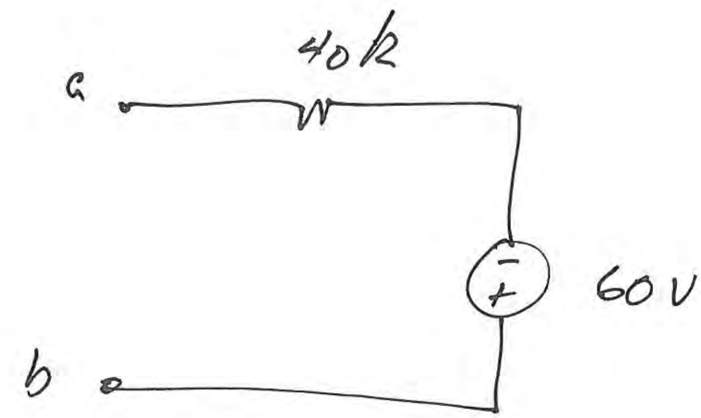
a
b
8k
160k
40k
$\frac{75V}{40k}$

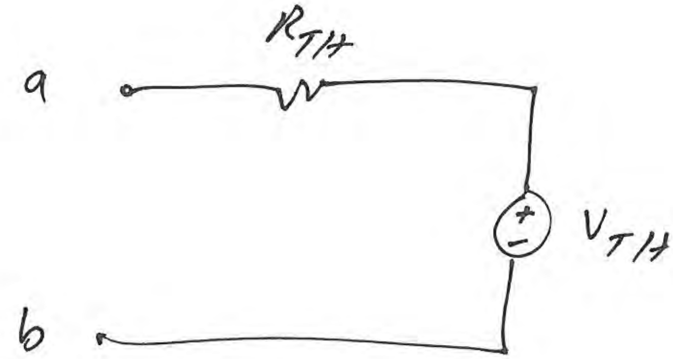


a
b
8k
32k
$\frac{75V}{40k}$



8+32k
$\frac{75V}{40k}$ 32k


40k
a
b
60 V


R_TH
a
b
V_TH

ALL DE'S OF FORM

$$\frac{dx}{dt} + \frac{x}{\tau} = K$$

FINAL VALUE OF x MUST BE CONSTANT

$$\Rightarrow \left.\frac{dx}{dt}\right|_{t\to\infty} = 0 \Rightarrow K = \frac{x_F}{\tau}$$

$$\frac{dx}{dt} + \frac{x}{\tau} = \frac{x_f}{\tau}$$

$$\frac{dx}{dt} = -\frac{(x-x_f)}{\tau}$$

INTEGRATE: $$\frac{dx}{(x-x_f)} = -\frac{dt}{\tau}$$

$$\left.\ln(x-x_f)\right|_{t_0}^{t} = -\frac{(t-t_0)}{\tau}$$

$$\ln\left[\frac{x(t)-x_f}{x(t_0)-x_f}\right] = -\frac{(t-t_0)}{\tau}$$

$\vdots$

$$x(t) = x_f + \left[x(t_0) - x_f\right]e^{-(t-t_0)/\tau}$$

FINAL VALUE

VALUE AT SWITCHING TIME, $t = t_0$

APPLICATION

1) ID VARIABLE OF INTEREST

RC CKT : CAPACITOR VOLTAGE
RL CKT : INDUCTOR CURRENT
WHY?

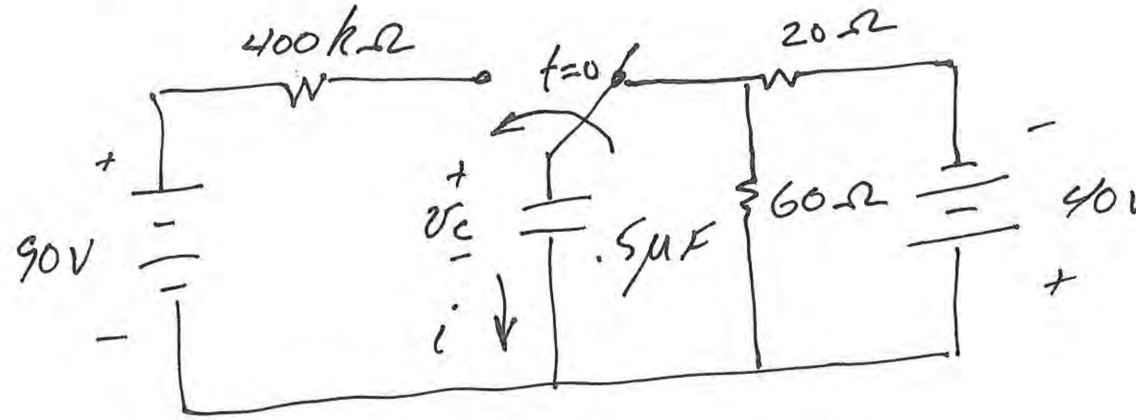
2) CALC VALUE OF VARIABLE AT $t = t_0$

WORRY ABOUT t_0^- OR t_0^+ ?

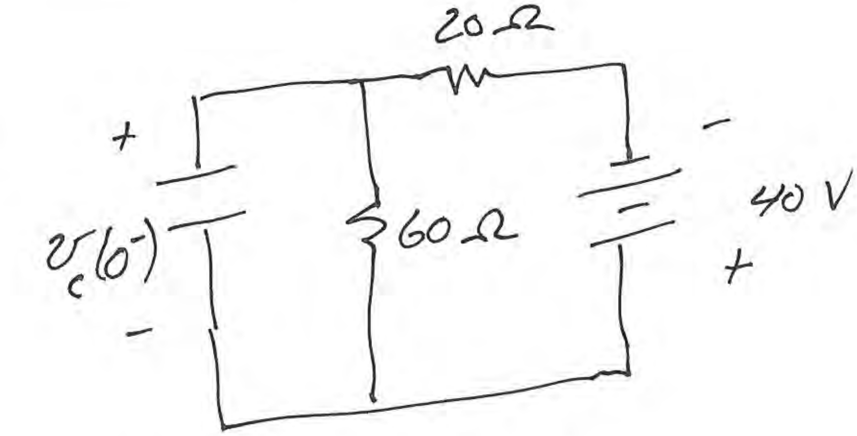
3) DETERMINE VALUE AS $t \to \infty$

4) CALCULATE TIME CONSTANT, τ

EXAMPLE 7.7



$t < 0$

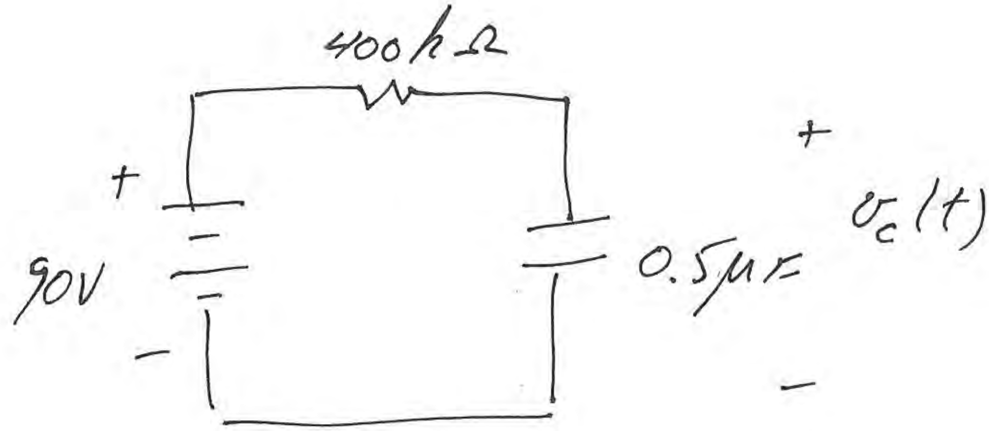


VOLTAGE DIV FOR $v_c(0^-)$:

$$v_c(0^-) = -40\left(\frac{60}{60+20}\right)$$

a) $\boxed{v_c(0^-) = -30\text{ V} = v_c(0^+)}$

$t \geq 0$



b) $\boxed{v_c(\infty) = 90\text{ V}}$

$$\tau = RC = (400\text{k}\Omega)(0.5\mu\text{F})$$

c) $\boxed{\tau = 0.25\text{ s}}$

$$v_c(t) = v_F + (v(0^+) - v_F)e^{-t/\tau}$$

$$= 90 + (-30 - 90)e^{-5t}\text{ V}$$

d) $\boxed{v_c(t) = 90 - 120e^{-5t}\text{ V}; \; t \geq 0}$

$$i = C\frac{dv_c(t)}{dt} = 0.5\mu F \frac{d}{dt}\left(90-120e^{-5t}\,V\right)$$

$$= (0.5\mu F)(600\,V/s)e^{-5t}$$

e) $\boxed{i(t) = 0.3e^{-5t}\,mA;\ t \geq 0^+}$

WHY $t \geq 0^+$?

ANS $i(0) = 0.3\,mA$, BUT

$i(0^-) = 0$

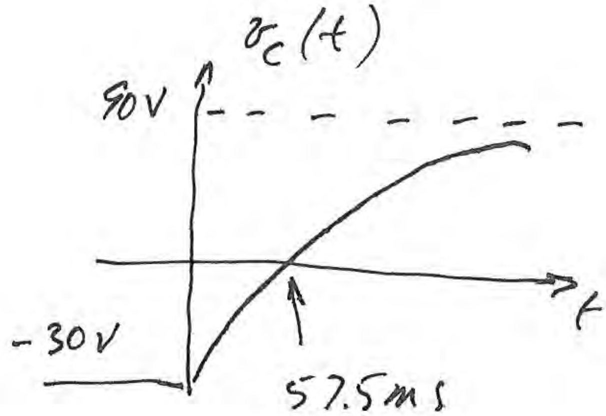
? TIME AT WHICH $v_c(t) = 0$

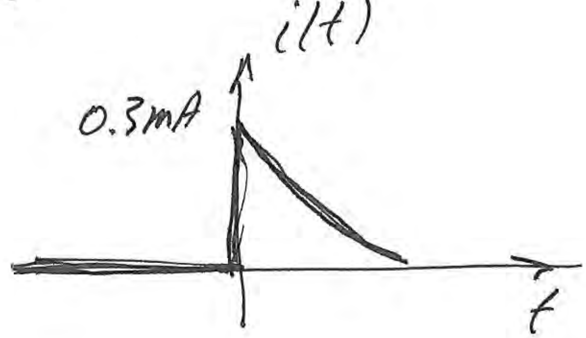
$$v_c(t_0) = 90 - 120e^{-5t_0} = 0$$

$$\frac{90}{120} = e^{-5t_0} \longrightarrow t_0 = \frac{\ln(90/120)}{-5}$$

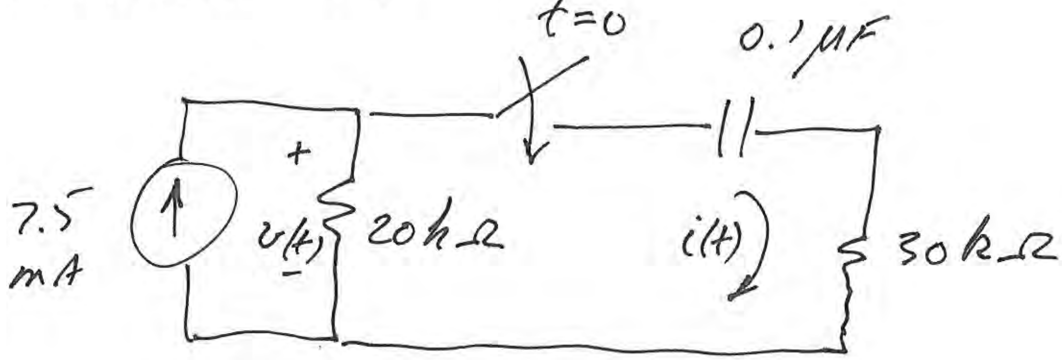
f) $\boxed{t_0 = 57.5\,ms}$

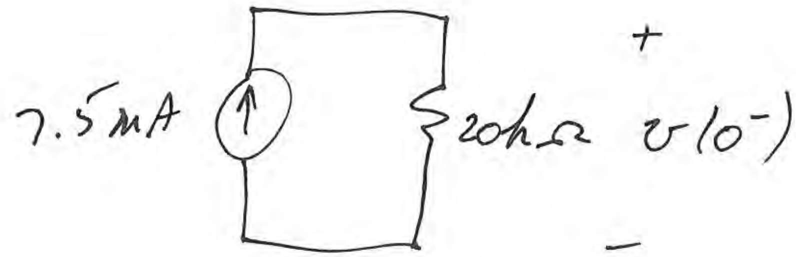
g)





EXAMPLE 7.8

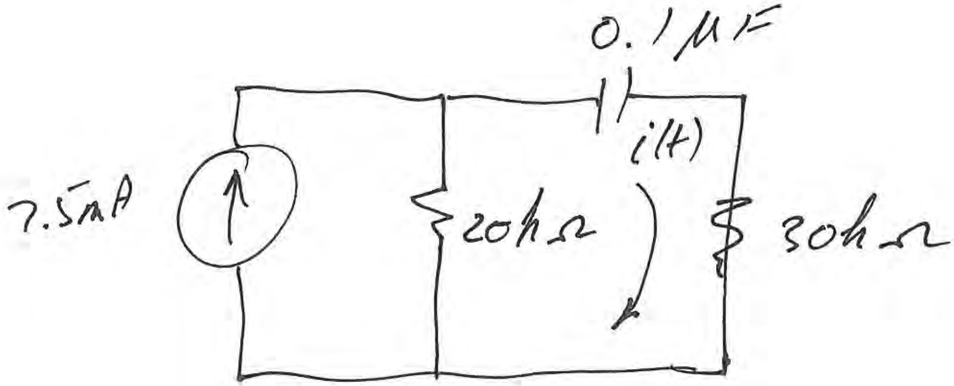




$t<0$

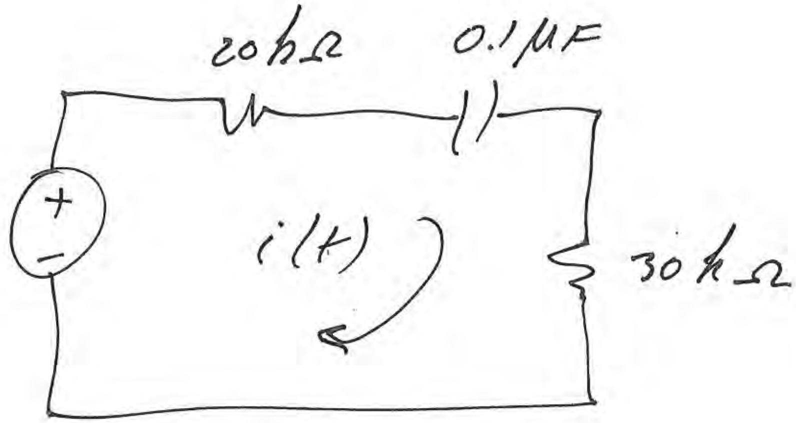
$$v(0^-) = (7.5\,mA)(20\,k\Omega)$$

$$\boxed{v(0^-) = 150\,V} \qquad \boxed{i(0^-) = 0}$$



$t \geq 0$

7.5 mA
× 20 kΩ
= 150 V



$$\tau = RC = (50\,k\Omega)(0.1\,\mu F)$$

$$\boxed{\tau = 5\times10^{-3}\,s}$$

RECALL $i(0^-) = 0$

$$i(0^+) = \frac{150\,V}{50\,k\Omega} = 3\,mA$$

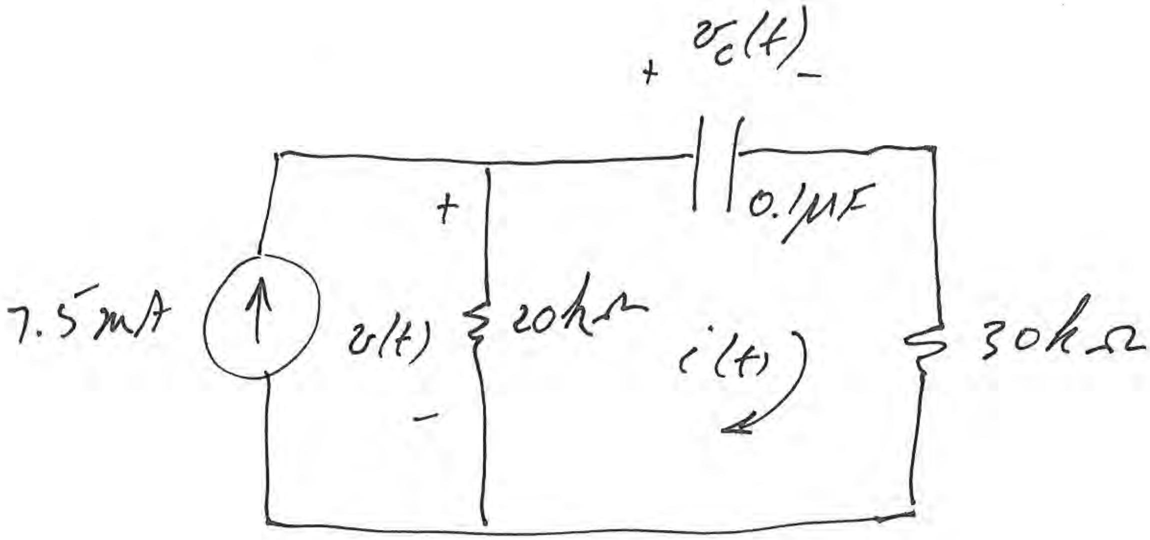
$$\boxed{i(0^+) = 3\,mA}$$

$$\boxed{i(\infty) = 0}$$

$$i(t) = i_f + \left[i(0^+) - i_f\right]e^{-200t}$$

$$\boxed{i(t) = 3e^{-200t}\ mA;\ t \geq 0^+}$$

WHY $t \geq 0^+$?



$$v_c(0^-) = 0 = v_c(0^+)$$

$$v(t) = v_c(t) + i(t)\,30k\Omega$$

$$i(t)\Big|_{t\to\infty} = 0 \Rightarrow v_c(\infty) = (7.5\,mA) \times (20k\Omega) = 150\ V$$

$$v_c(t) = v_f + [v_c(0^+) - v_f]\,e^{-t/\tau}$$

$$v_c(t) = 150 + (0-150)\,e^{-200t}\ V$$

$$v_c(t) = 150(1-e^{-200t})\,V; \quad t \geq 0$$

WHY $t \geq 0$?

$$v(t) = v_c(t) + i(t)\,30k\Omega$$

$$= 150(1-e^{-200t})V + (30k\Omega)(3mA\,e^{-200t})$$

$$= 150 - 150e^{-200t} + 90e^{-200t}\ V$$

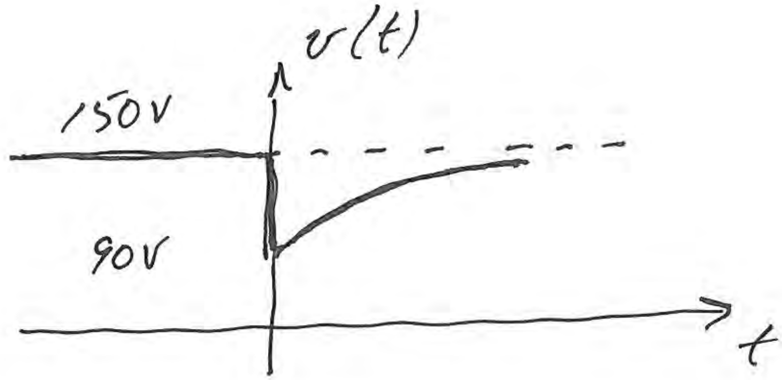
$$\boxed{v(t) = 150 - 60e^{-200t}\ V\ ;\ t \geq 0^+}$$

WHY $t \geq 0^+$?

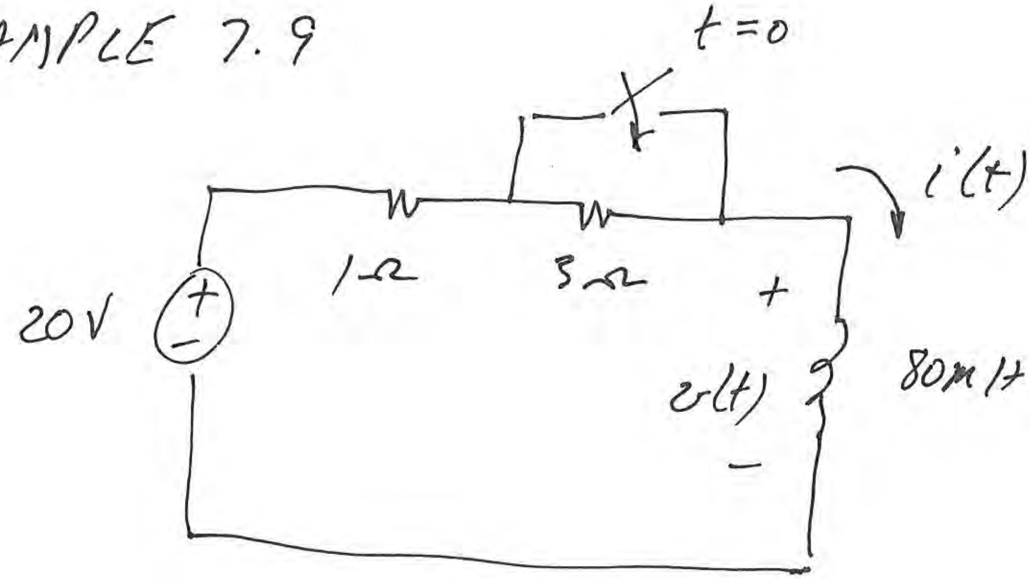
ANS: SOL'N GIVES $v(0) = 150-60$

$= 90V$

BUT RECALL $v(0^-) = 150V$



EXAMPLE 7.9



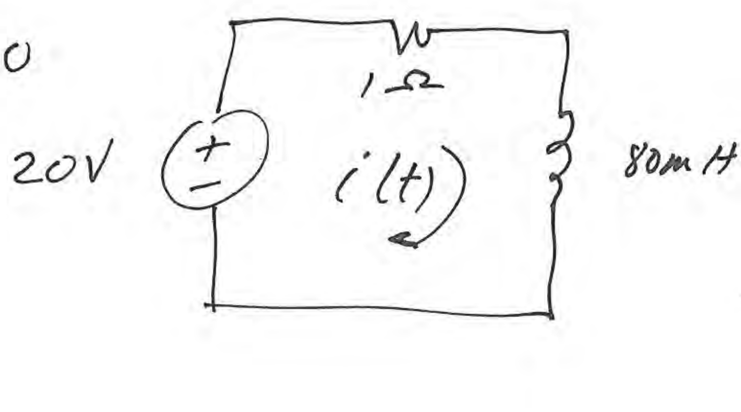
WANT: $v(t)$; $t \geq 0^+$

$i(t)$; $t \geq 0$

$v(0^-) = 0$

$i(0^-) = \dfrac{20V}{4\,\Omega} = 5A$

$t \geq 0$



$i(\infty) = \dfrac{20V}{1\,\Omega} = 20A$

$\tau = L/R = \dfrac{80\,mH}{1\,\Omega}$

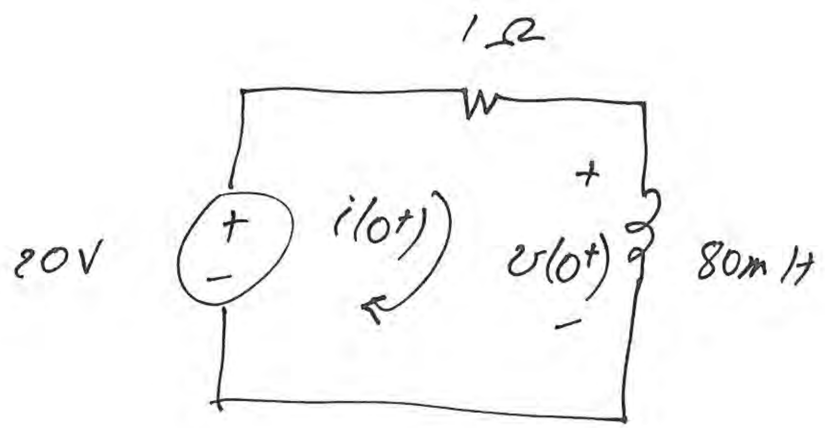
$\tau = 80\,mS$

$1/\tau = 12.5\,S^{-1}$

$$i(t) = i_f + [i(0^+) - i_f]e^{-t/\tau}$$

$$= 20 + (5-20)e^{-12.5t} \text{ A}$$

$$\boxed{i(t) = 20 - 15e^{-12.5t} \text{ A}; \; t \geq 0}$$



$t = 0^+$

LOOP EQ: $20\text{V} - i(0^+)(1\Omega) - v(0^+) = 0$

$$20 - 5\text{A}(1\Omega) = v(0^+) = 15\text{V}$$

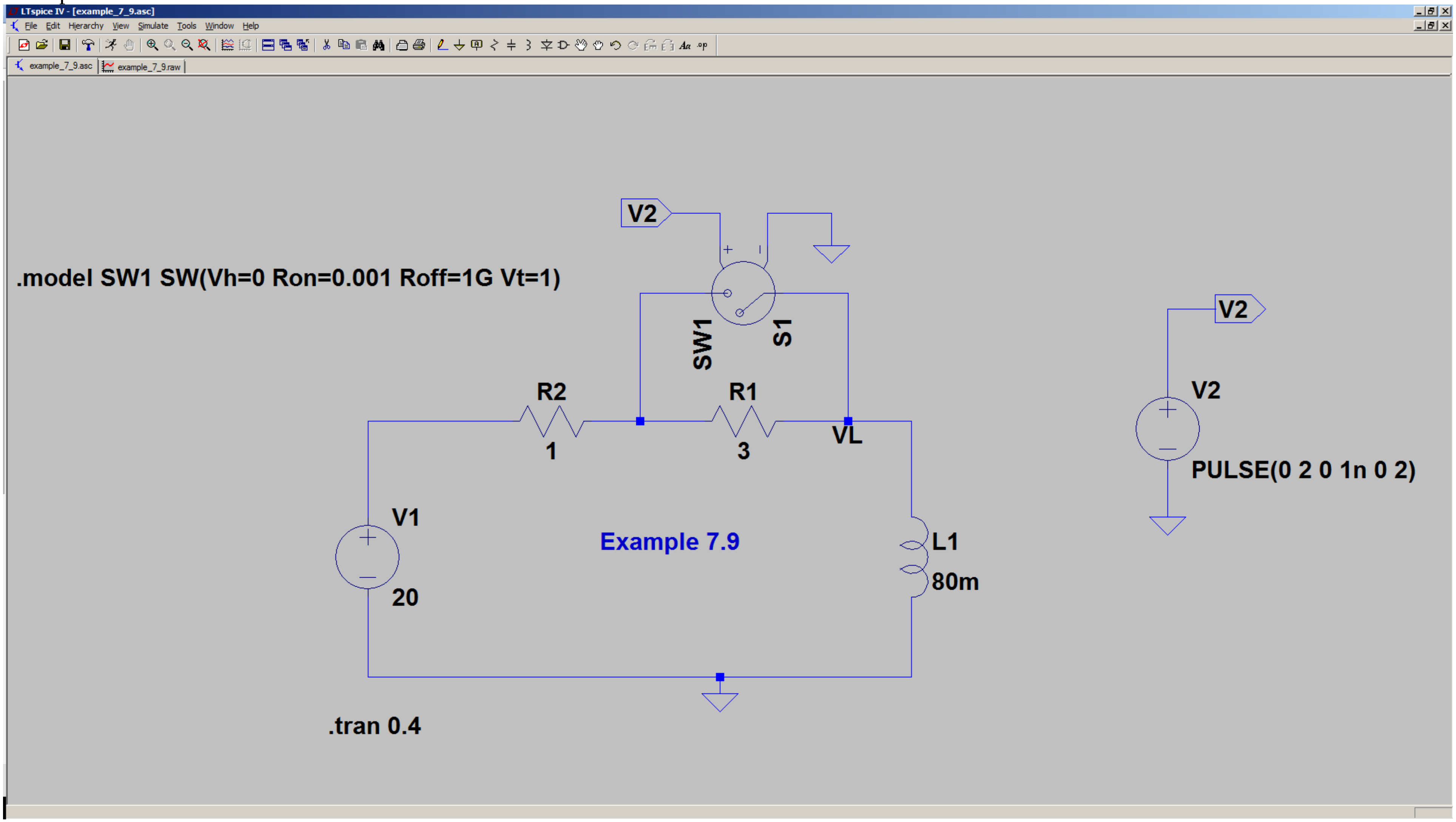
$$v(\infty) = 0$$

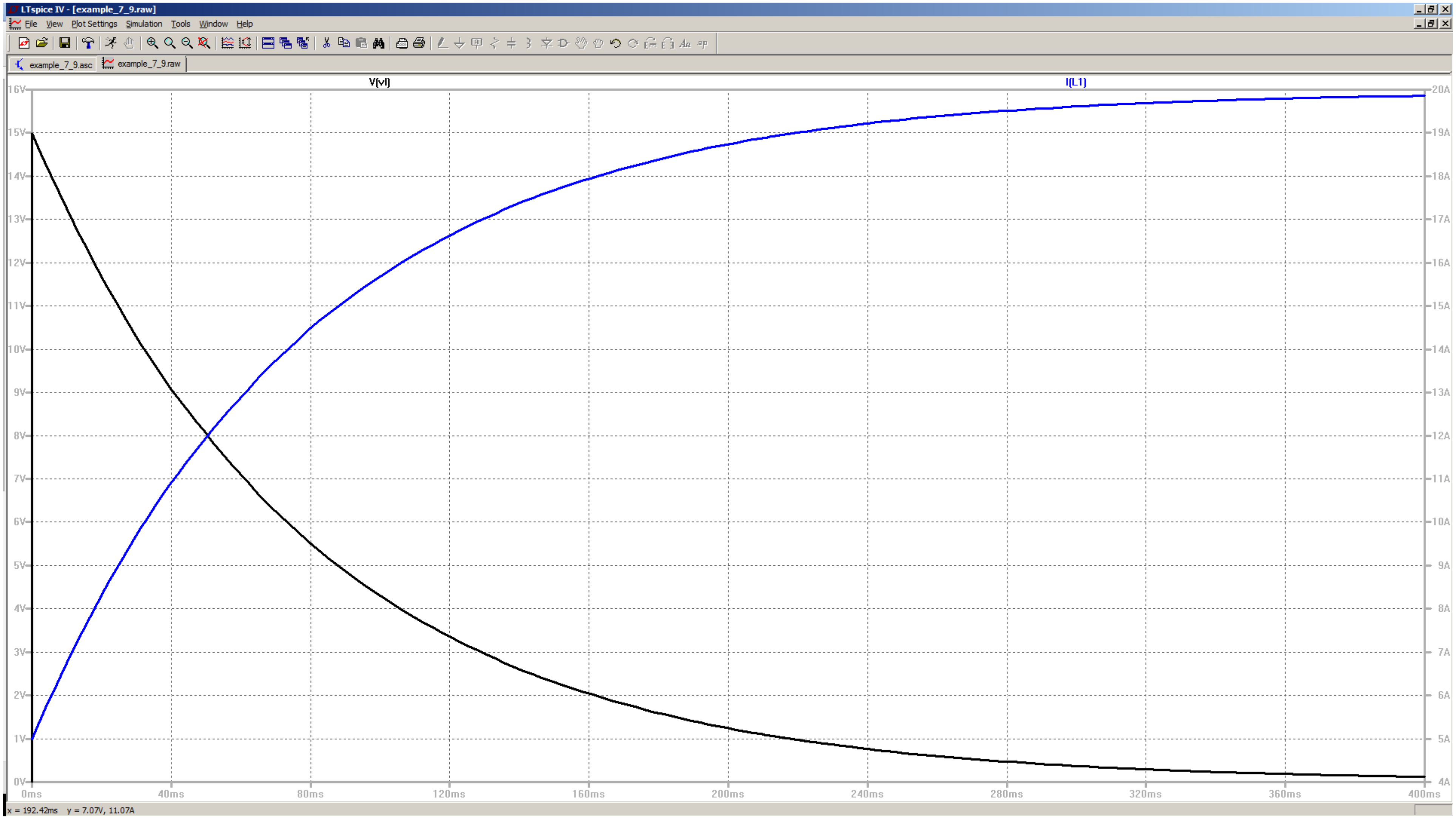
$$v(t) = v_f + [v(0^+) - v_f]e^{-t/\tau}$$

$$\boxed{v(t) = 15e^{-12.5t} \text{ V}; \; t \geq 0^+}$$

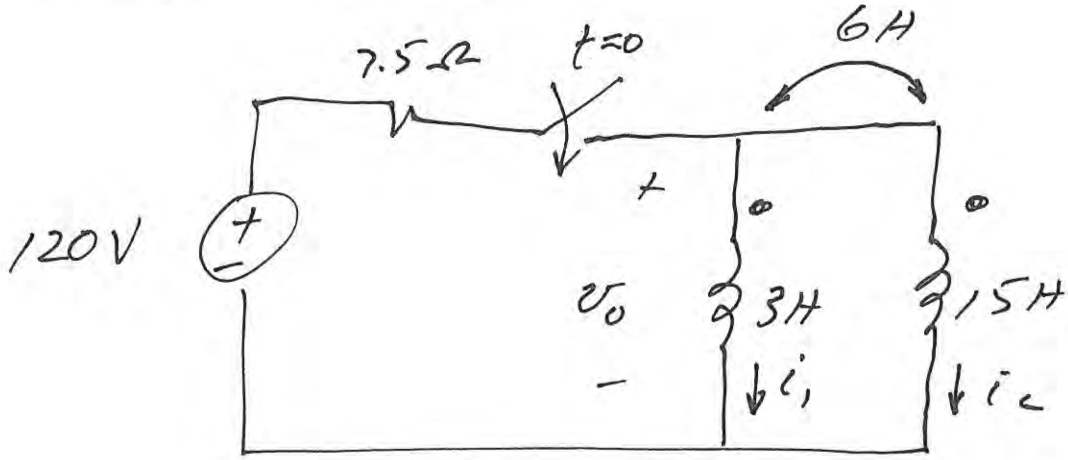
WHY $t \geq 0^+$?

Example 7.9


LTspice IV - [example_7_9.asc]
File Edit Hierarchy View Simulate Tools Window Help
example_7_9.asc
example_7_9.raw
.model SW1 SW(Vh=0 Ron=0.001 Roff=1G Vt=1)
V2
SW1
S1
R2
1
R1
3
VL
V1
20
Example 7.9
L1
80m
V2
V2
PULSE(0 2 0 1n 0 2)
.tran 0.4

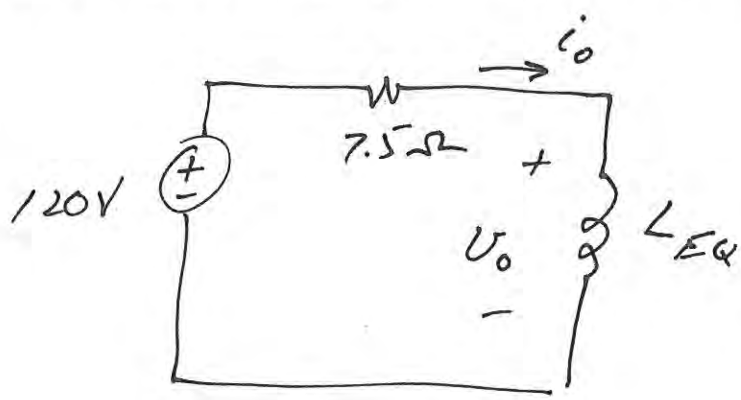

LTspice IV - [example_7_9.raw]
File View Plot Settings Simulation Tools Window Help
example_7_9.asc
example_7_9.raw
V(vl)
I(L1)
16V
15V
14V
13V
12V
11V
10V
9V
8V
7V
6V
5V
4V
3V
2V
1V
0V
20A
19A
18A
17A
16A
15A
14A
13A
12A
11A
10A
9A
8A
7A
6A
5A
4A
0ms
40ms
80ms
120ms
160ms
200ms
240ms
280ms
320ms
360ms
400ms
x = 192.42ms y = 7.07V, 11.07A

EXAMPLE 7.10



NO INITIAL STORED ENERGY

$t \geq 0$



$$L_{EQ} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} = 1.5H$$

$$\tau = L/R = \frac{1.5H}{7.5\,\Omega} = 0.2S$$

$$i_o(0^-) = 0 = i_o(0^+)$$

$$i_o(\infty) = \frac{120V}{7.5\,\Omega} = 16A$$

$$i_o(t) = i_f + [i_o(0^+) - i_f]e^{-t/\tau}$$

$$\boxed{i_o(t) = 16(1 - e^{-5t})A \; ; \; t \geq 0}$$

$$v_o(t) = L_{EQ}\frac{di_o(t)}{dt}$$

$$= 1.5H \frac{d}{dt}\left[16(1-e^{-5t})A\right]$$

$$= (1.5H)(-16A)(-5/s)e^{-5t}$$

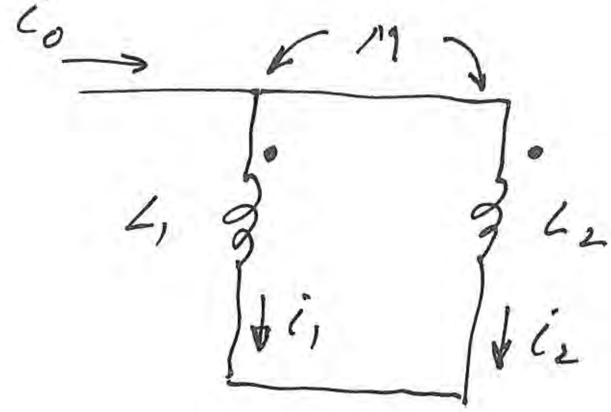
$$\boxed{v_o(t) = 120e^{-5t} \; ; \; t \geq 0^+}$$

WHY $t \geq 0^+$?

FROM SOL'N, $v_o(0) = 120V$

BUT $v_o(0^-) = 0V$

(DISCONTINUOUS AT $t=0$)



$$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\frac{di_1}{dt}(L_1 - M) = \frac{di_2}{dt}(L_2 - M)$$

$$\frac{di_1}{dt} = -3 \frac{di_2}{dt} \quad (A)$$

$$i_0 = i_1 + i_2 \quad (B)$$

$$(A) \,\&\, (B) \Rightarrow \frac{di_2}{dt} = -\frac{1}{2} \frac{di_0}{dt}$$

$$\frac{di_2}{dt} = -\frac{1}{2} \frac{d}{dt}\left[16(1 - e^{-5t})A\right]$$

$$\frac{di_2}{dt} = -40e^{-5t} \text{ A/s}$$

$$di_2 = -40e^{-5t}dt$$

$$\int_0^t di_2 = \int_0^t (-40e^{-5x}dx)$$

$$i_2(t) - i_2(0) = \frac{-40}{-5}e^{-5x}\Big|_0^t = 8(e^{-5t}-1)$$

$$i_2(t) = 8(e^{-5t}-1) + \underbrace{i_2(0)}_{=0}$$

$$\boxed{i_2(t) = 8(e^{-5t}-1)A;\ t \geq 0}$$

FROM $i_0(t) = i_1(t) + i_2(t)$

$$\boxed{i_1(t) = 24(1-e^{-5t})A;\ t \geq 0}$$

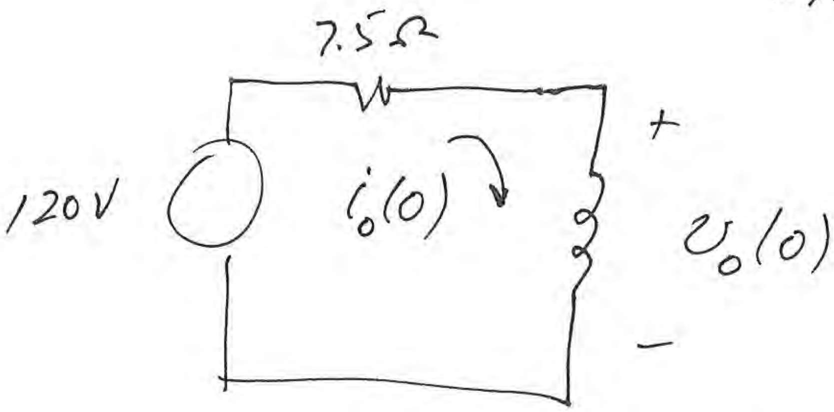
RECAP

$$i_o(t) = 16(1-e^{-5t})A \; ; \; t \geq 0$$

$$i_1(t) = 24(1-e^{-5t})A \; ; \; t \geq 0$$

$$i_2(t) = -8(1-e^{-5t})A \; ; \; t \geq 0$$

$i_o(0) = i_1(0) = i_2(0)$ CONSISTENT WITH NO STORED INITIAL ENERGY



SOLUTION FOR $v(t)$ GIVES $v(0) = 120V$

(CONSISTENT WITH $i_o(0) = 0$)

IS v_0 CONSISTENT WITH i_1 & i_2?

$$v_0(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$= 3H \frac{d}{dt}\left[24(1-e^{-5t})A\right]$$

$$+6H \frac{d}{dt}\left[-8(1-e^{-5t})A\right]$$

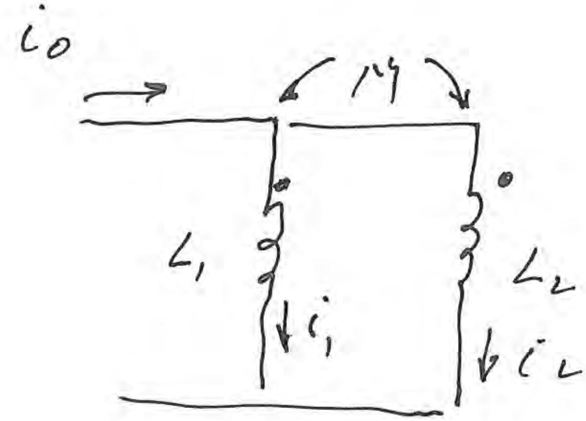
$$\vdots$$

$$= 120 e^{-5t} V ; \ t \geq 0^+ \quad \underline{\underline{OK}}$$

WHAT ABOUT FINAL VALUES OF i_1 & i_2?

$$i_1(\infty) = 24A$$

$$i_2(\infty) = -8A$$



$\underline{\underline{\text{PROBLEM}}}$

PROBLEM: AS $t \to \infty$ BOTH INDUCTORS BECOME IDEAL SHORT CIRCUITS.

$\Rightarrow$ IT'S IMPOSSIBLE TO ESTABLISH FINAL VALUES USING CURRENT DIV.

$\longrightarrow$ WE MUST CONSIDER FLUX LINKAGES

FLUX LINKING 3H COIL (λ_1) MUST EQUAL FLUX LINKING 15H COIL (λ_2) BECAUSE

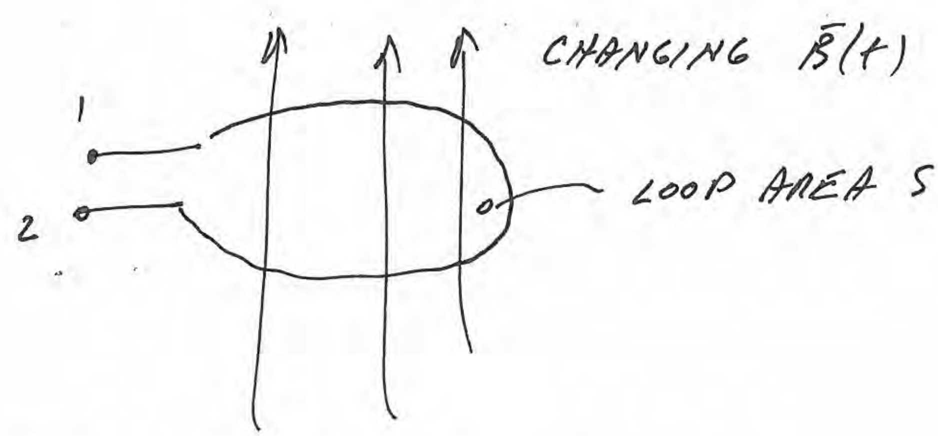
$$v_o = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt} \quad \text{WHY?}$$

NOW $\lambda_1 = L_1 i_1 + M i_2 = 3i_1 + 6i_2$ Wb-TURNS

$\lambda_2 = L_2 i_2 + M i_1 = 15 i_2 + 6 i_1$ Wb-TURNS

L & M IN HENRYS, $H = \frac{\text{Wb}}{\text{A}} = \frac{\text{V}\cdot\text{S}}{\text{A}}$

FARADAY'S LAW



$\bar{B}(t)$ = MAGNETIC FLUX DENSITY

$$[\bar{B}(t)] = \frac{Wb}{m^2} = \text{TESLA}$$

$Wb = V \cdot S$

INDUCED VOLTAGE

$$V_{12} = -N \int_S \frac{\partial \bar{B}(t)}{\partial t} \cdot d\bar{s}$$

= (# TURNS) × (MAG FLUX TIME RATE OF CHANGE) × AREA

$$v = \frac{d\lambda}{dt} \qquad \underbrace{\lambda = N B(t) S}_{\text{FLUX LINKAGE}}$$

$[\lambda] = V \cdot S$

USE SOL'NS FOR $i_1(t)$ & $i_2'(t)$:

$$\lambda_1 = 24(1-e^{-5t}) \text{ Wb-TURNS}$$

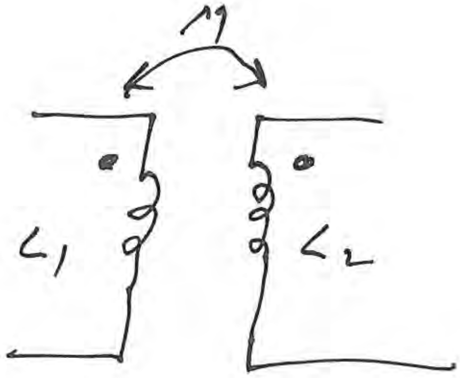
$$\lambda_2 = 24(1-e^{-5t}) \text{ Wb-TURNS}$$

$$\Rightarrow \lambda_1(\infty) = \lambda_2(\infty) = 24 \text{ Wb-TURNS}$$

FINAL VALUE OF FLUX LINKING EITHER COIL IS SAME

$\Rightarrow$ $i_1(\infty)$ & $i_2'(\infty)$ ARE CONSISTENT

MAGNETICALLY COUPLED INDUCTORS



M = MUTUAL INDUCTANCE

$$M = k\sqrt{L_1 L_2}$$

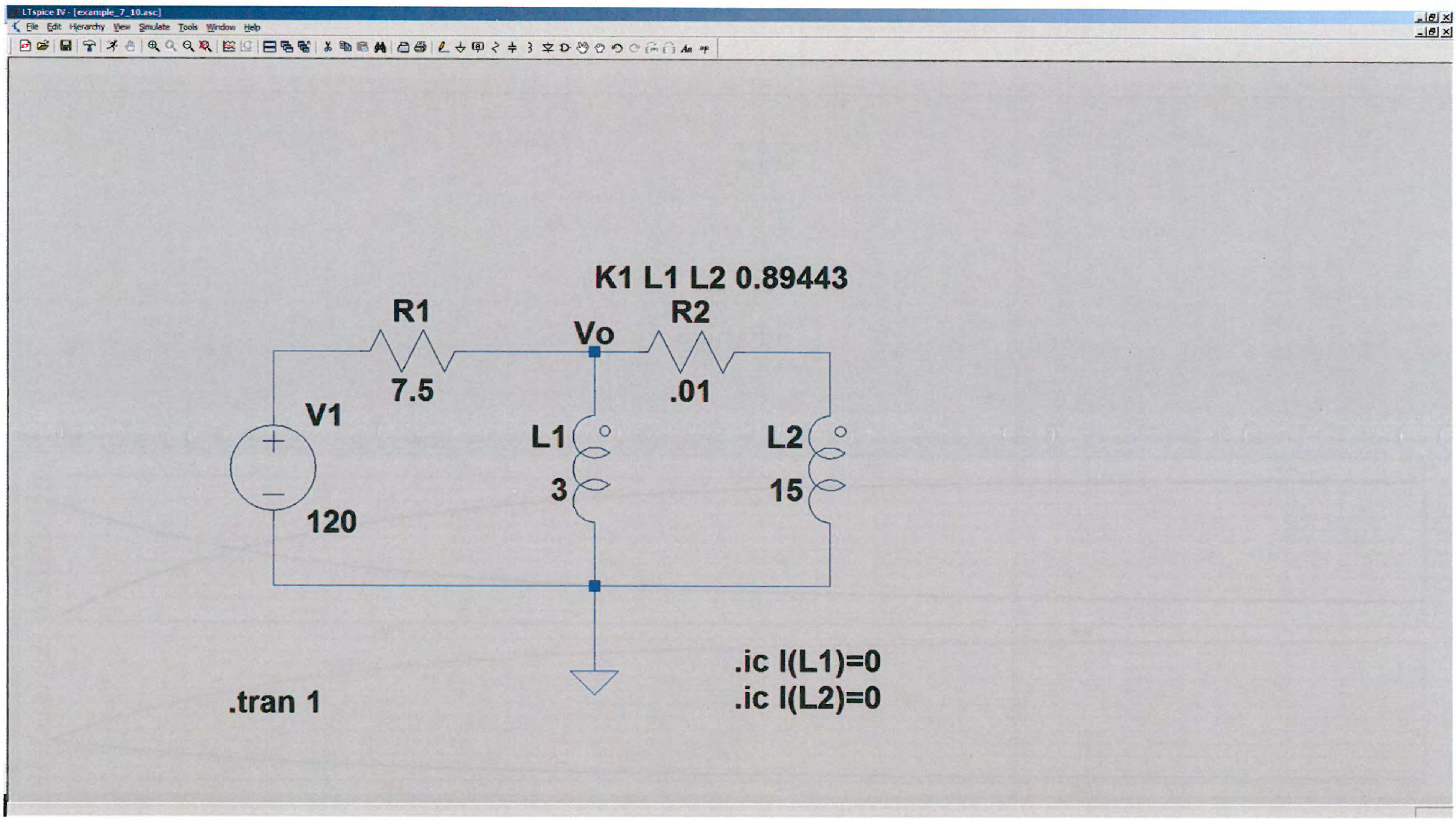
COUPLING COEFFICIENT

TO COUPLE INDUCTORS IN LTSPICE
USE A "K" DIRECTIVE

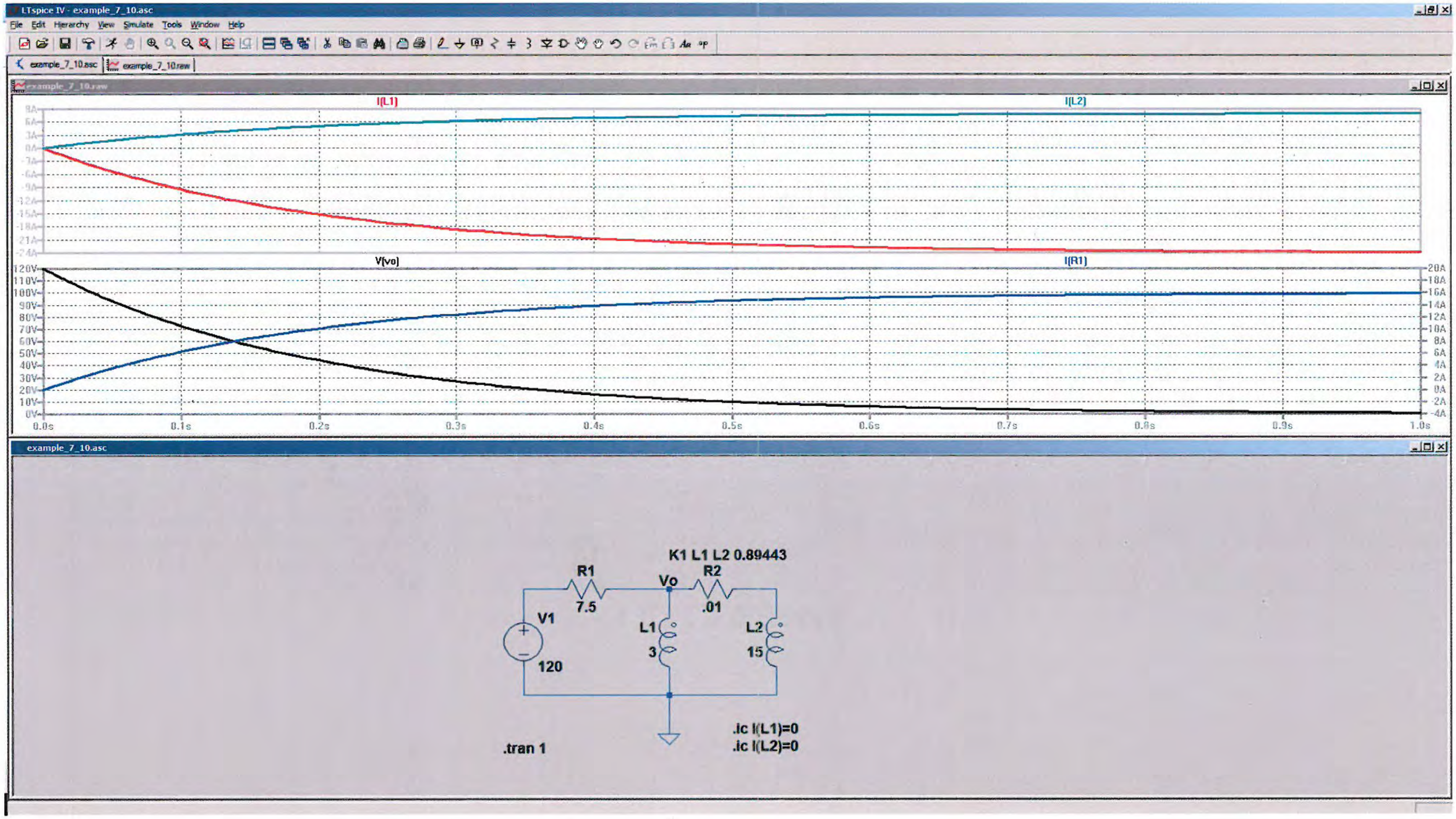
$|k| \leq 1$

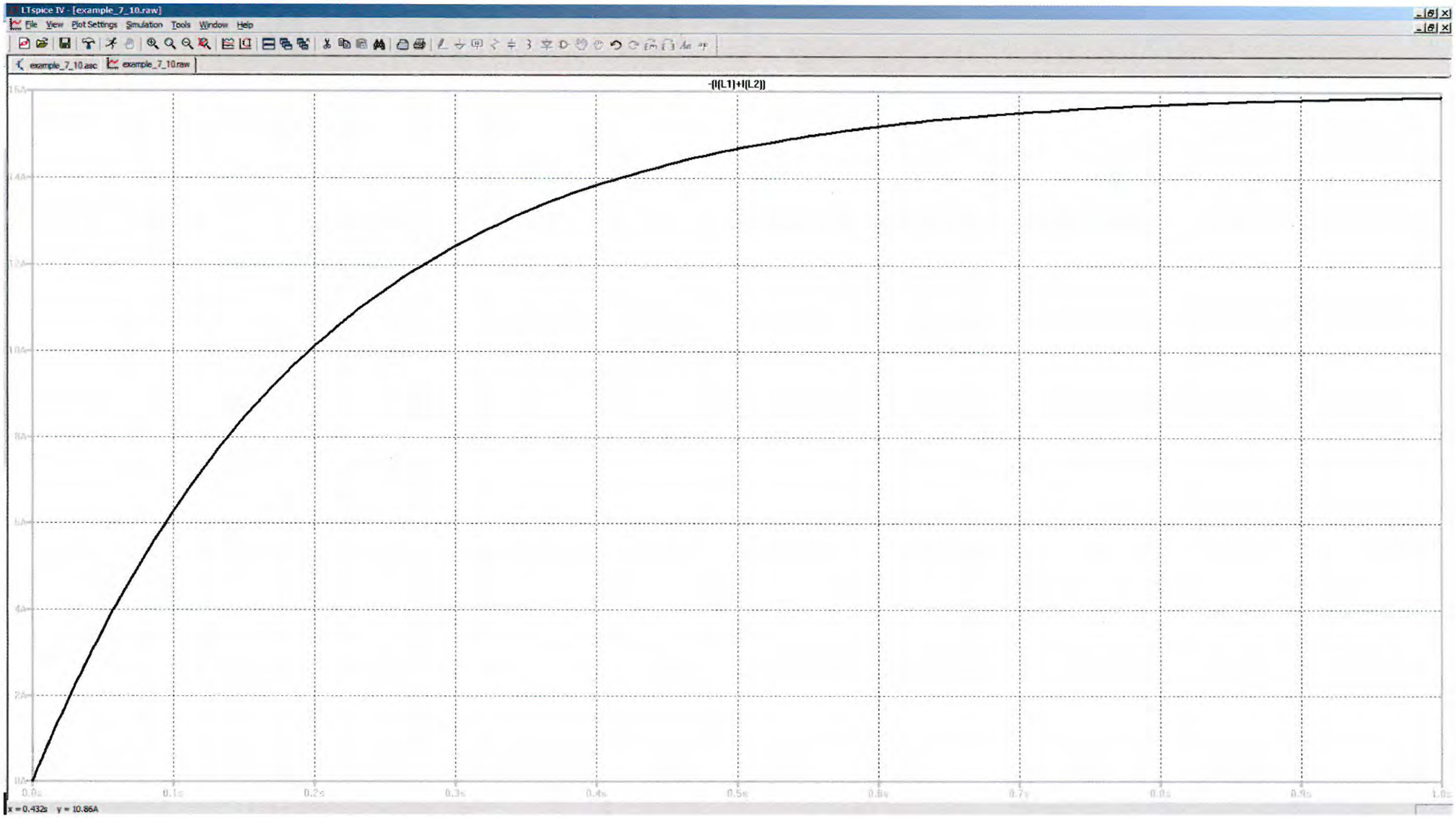
SYNTAX: K L1 L2 k

Example 7.10



LTspice IV - [example_7_10.asc]
File Edit Hierarchy View Simulate Tools Window Help
K1 L1 L2 0.89443
R1
7.5
Vo
R2
.01
V1
120
L1
3
L2
15
.tran 1
.ic I(L1)=0
.ic I(L2)=0


LTspice IV - example_7_10.asc
File Edit Hierarchy View Simulate Tools Window Help
example_7_10.asc
example_7_10.raw
I(L1)
I(L2)
V(vo)
I(R1)
0.0s
0.1s
0.2s
0.3s
0.4s
0.5s
0.6s
0.7s
0.8s
0.9s
1.0s
120V
110V
100V
90V
80V
70V
60V
50V
40V
30V
20V
10V
0V
20A
18A
16A
14A
12A
10A
8A
6A
4A
2A
0A
-2A
-4A
K1 L1 L2 0.89443
R1
7.5
Vo
R2
.01
V1
120
L1
3
L2
15
.tran 1
.ic I(L1)=0
.ic I(L2)=0


LTspice IV - [example_7_10.raw]
File View Plot Settings Simulation Tools Window Help
example_7_10.asc
example_7_10.raw
-(I(L1)+I(L2))
x = 0.432s y = 10.86A

Example 7.11 Analyzing an *RL* Circuit that has Sequential Switching

The two switches in the circuit shown in Fig. 7.31 have been closed for a long time. At $t = 0$, switch 1 is opened. Then, 35 ms later, switch 2 is opened.

a) Find $i_L(t)$ for $0 \le t \le 35$ ms.

b) Find i_L for $t \ge 35$ ms.

c) What percentage of the initial energy stored in the 150 mH inductor is dissipated in the 18 Ω resistor?

d) Repeat (c) for the 3 Ω resistor.

e) Repeat (c) for the 6 Ω resistor.

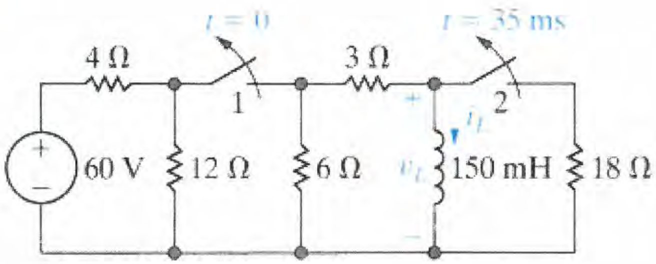


Figure 7.31 ▲ The circuit for Example 7.11.

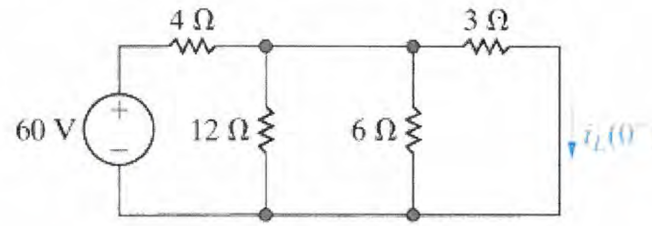


Figure 7.32 ▲ The circuit shown in Fig. 7.31, for $t < 0$.

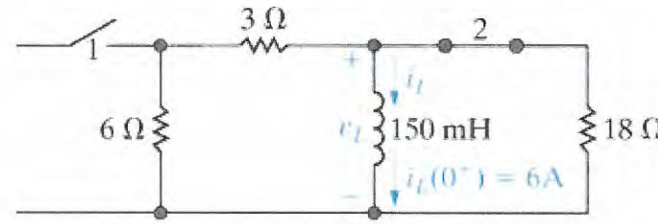


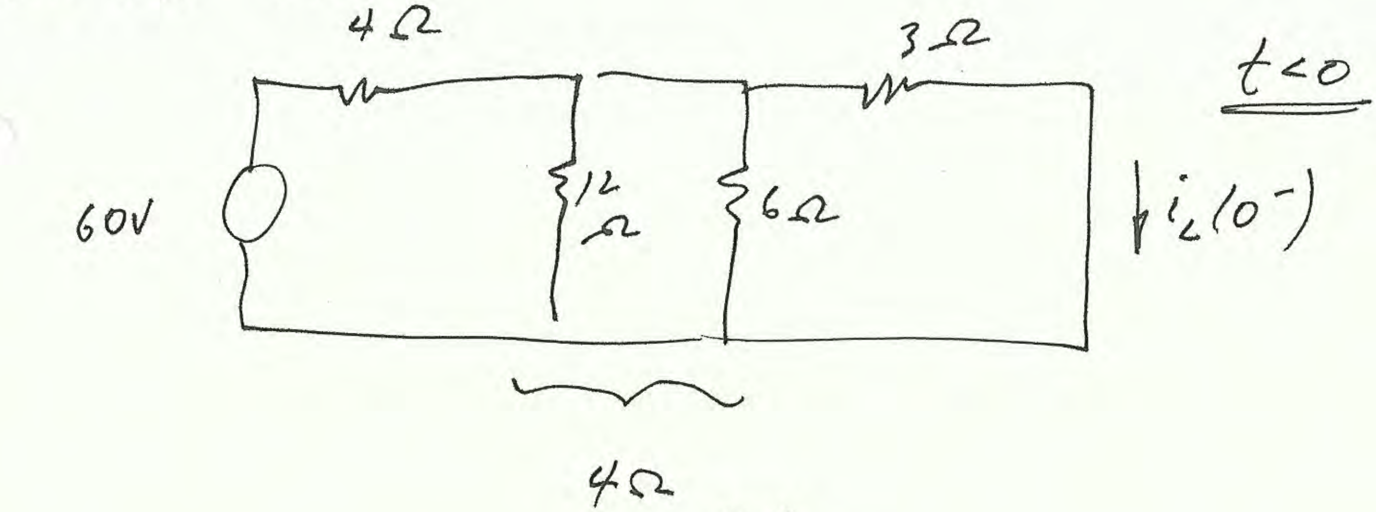
Figure 7.33 ▲ The circuit shown in Fig. 7.31, for $0 \le t \le 35$ ms.

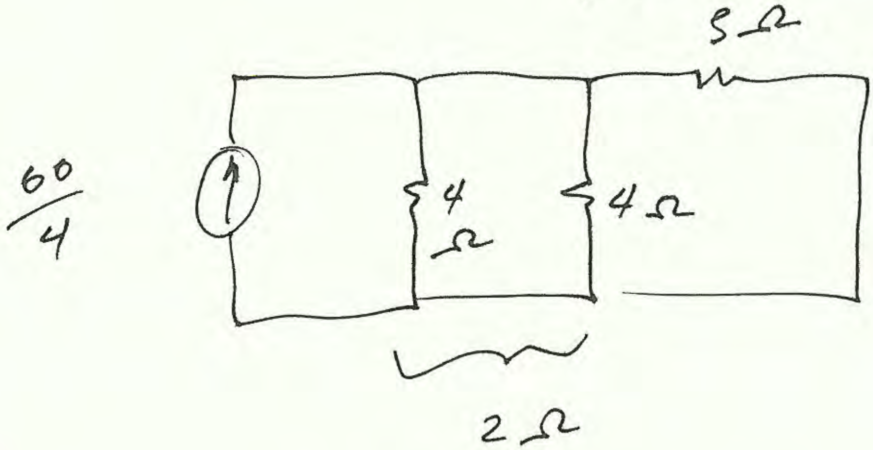
b) When $t = 35$ ms, the value of the inductor current is

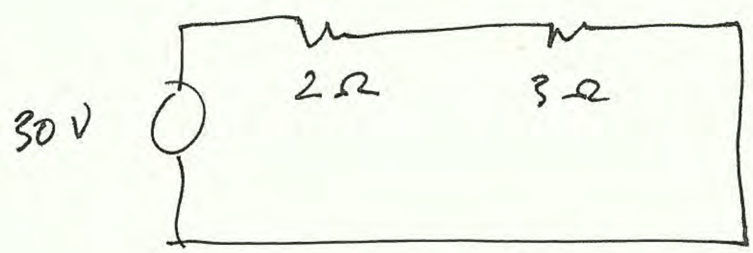
$$i_L = 6e^{-1.4} = 1.48 \text{ A}.$$

Thus, when switch 2 is opened, the circuit reduces to the one shown in Fig. 7.34, and the

Ex 7.11 PG 237

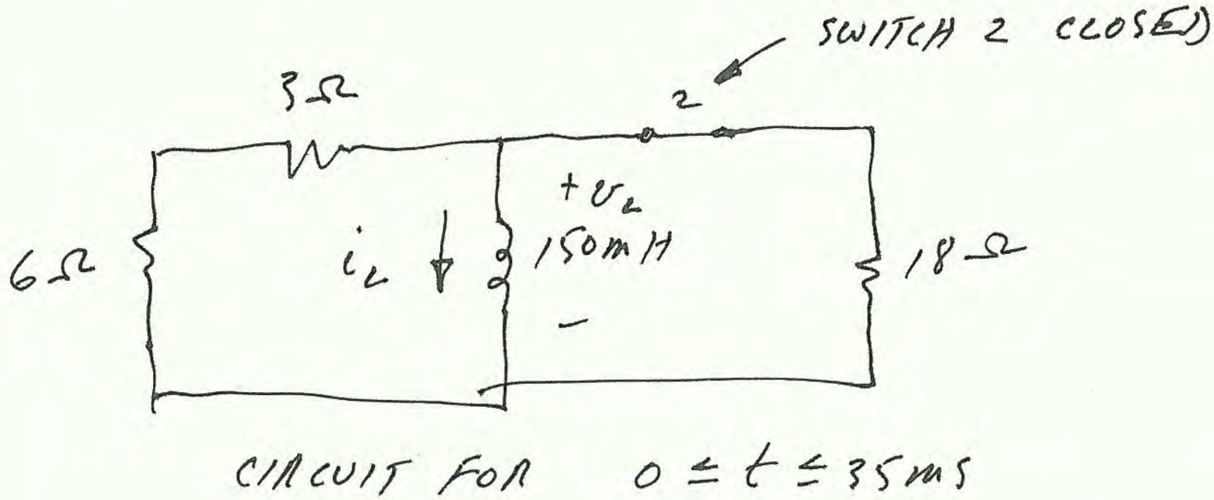




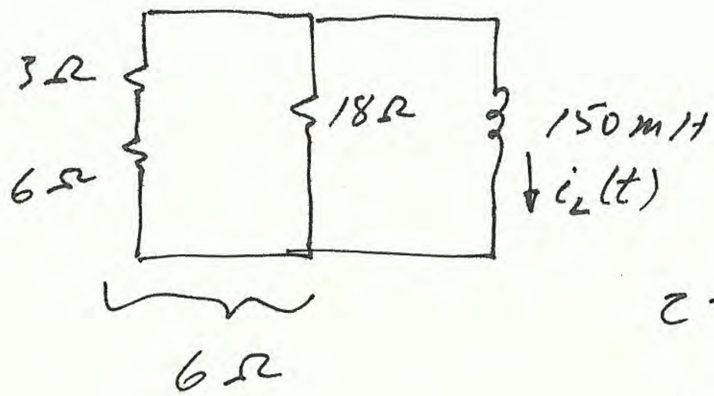


$i_L(0^-) = \frac{30V}{5\Omega} = 6A$

$i_L(0^+) = 6A$



CIRCUIT FOR $0 \le t \le 35ms$



$$\tau = L/R = \frac{150\,mH}{6\,\Omega}$$

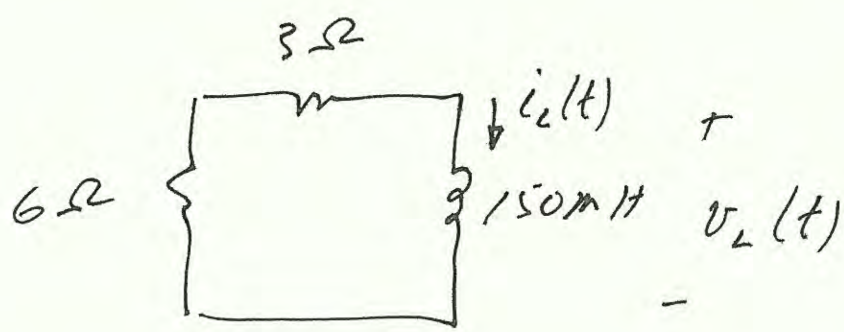
$$= 25\,ms$$

$$1/\tau = 40\,s^{-1}$$

$$i_L(t) = 6e^{-40t}\,A; \quad 0 \le t \le 35\,ms \quad \bigstar$$

(t EXPRESSED IN s)

AT $t = 35\,ms \equiv t_{sw}$ CKT BECOMES



WHAT IS $i_L(t)\big|_{t=t_{sw}^-}$?

FROM $\bigstar$ $i_L(t_{sw}^-) = i_L(t_{sw}^+) = 6e^{-40t}\Big|_{t=35ms}$

$$i_L(t_{sw}^+) = 1.48A$$

NOW WE HAVE A NEW TIME CONSTANT

$$\tau = L/R = \frac{150\,mH}{9\,\Omega} = 16.67\,ms$$

$$1/\tau = 60\,s^{-1}$$

$$i_L(t) = 1.48\,e^{-(t-0.035)60}\ A$$

$$35\,ms \le t$$

NOTE EXPONENT $=0$ AT SWITCHING TIME

AT THIS POINT $(t = t_{sw})$

$$i_L(t_{sw}) = 1.48\,A$$

$$i_L(t) = 6\exp\left(-\frac{t}{\tau_1}\right); \quad 0 \le t \le t_{sw}$$

$$\tau_1 = 25\text{ms}; \quad t_{sw} = 35\text{ms}$$

$$i_L(t) = 6\exp(-40t); \quad 0 \le t \le t_{sw}$$

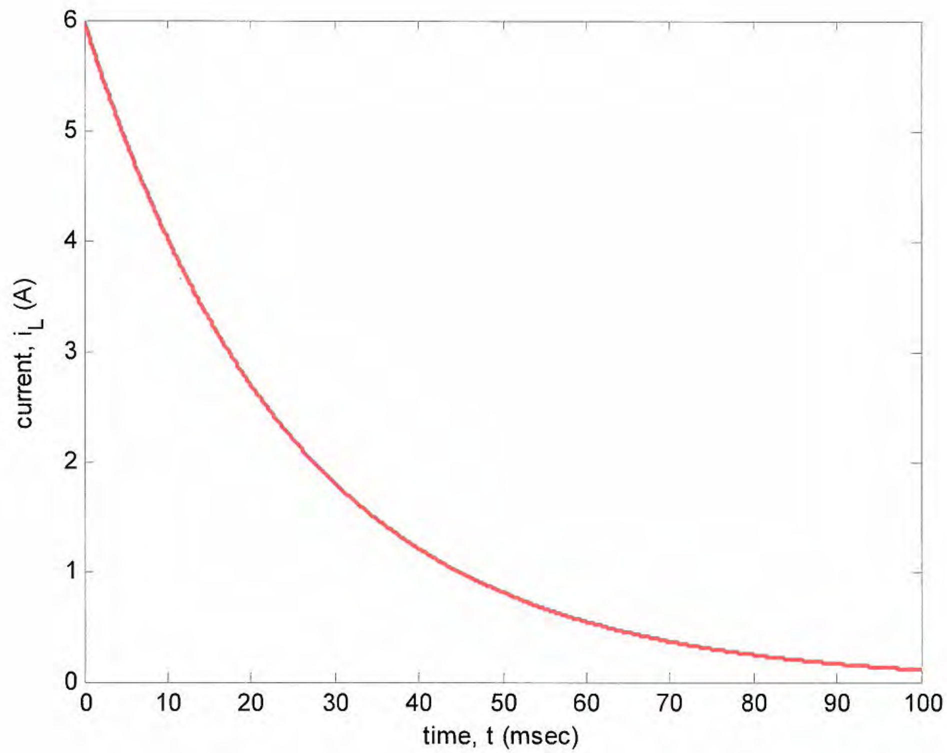
$$(\text{time expressed in sec})$$

$$i_L(t) = 6\exp(-0.04t); \quad 0 \le t \le t_{sw}$$

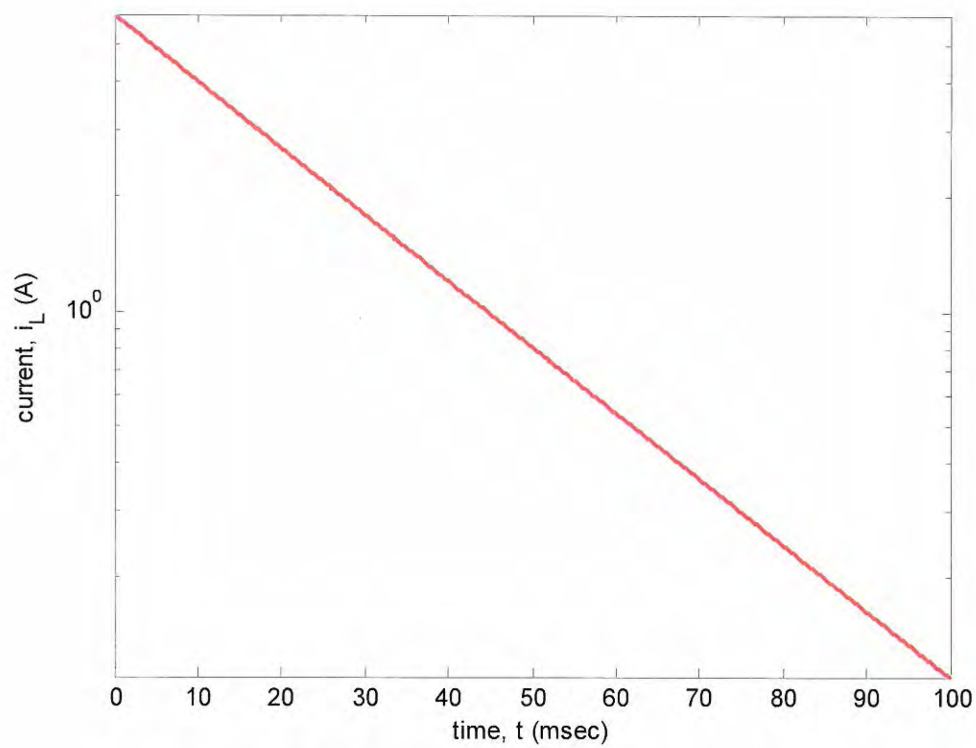
$$(\text{time expressed in msec})$$

$$i_L(t) = 6\exp\left(-\frac{t_{sw}}{\tau_1}\right)\exp\left[\frac{(t-t_{sw})}{\tau_2}\right]; \quad t_{sw} \le t$$

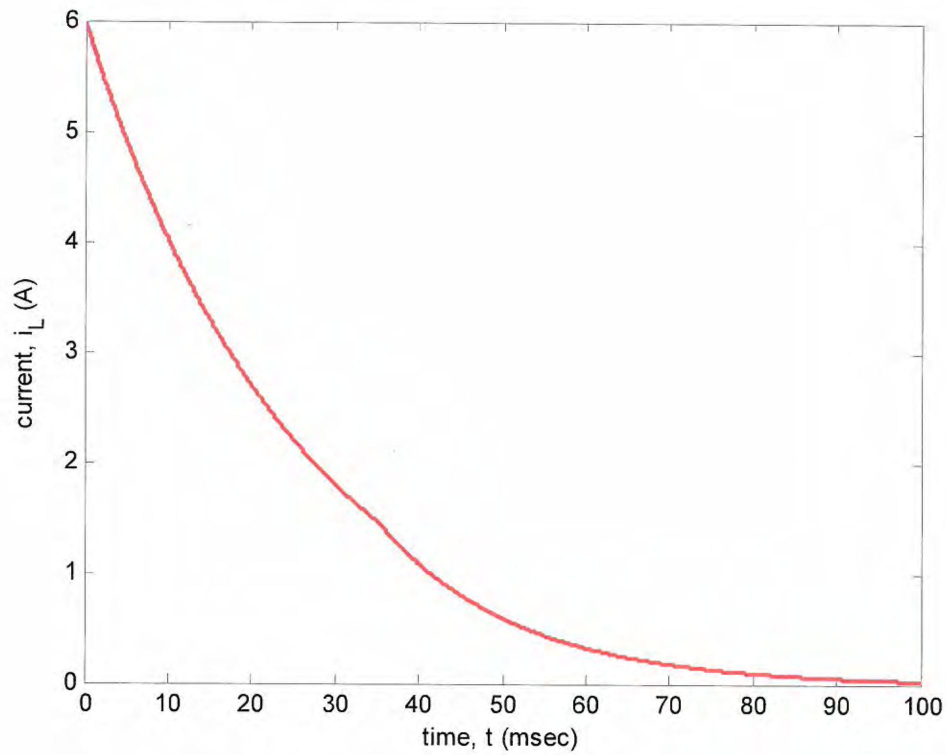
$$\tau_1 = 25\text{ms}; \quad \tau_2 = 16.67\text{ms}; \quad t_{sw} = 35\text{ms}$$

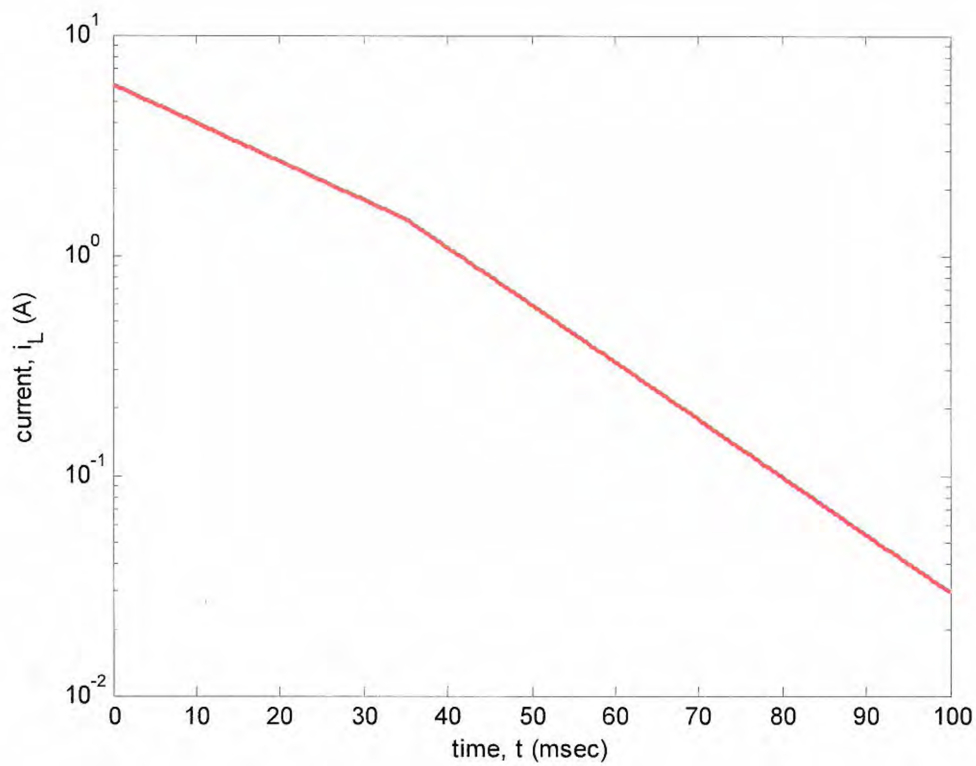


current, i_L (A)
time, t (msec)
0
1
2
3
4
5
6
10
20
30
40
50
60
70
80
90
100



current, i_L (A)
10^0
0
10
20
30
40
50
60
70
80
90
100
time, t (msec)


current, i_L (A)
0
1
2
3
4
5
6
time, t (msec)
0
10
20
30
40
50
60
70
80
90
100



10^{1}
10^{0}
10^{-1}
10^{-2}
current, i_L (A)
0
10
20
30
40
50
60
70
80
90
100
time, t (msec)

```
% example 7_11 in text
% 01/17/14 D D Duncan
N = 1000;
tmax = 100;% max time in msec
t = linspace(0,tmax, N);% time in msec
tau1 = 25;% time constant in msec
i_L_1 = 6*exp(-t/tau1);% current in amperes
figure(1); plot(t,i_L_1,'r-');
xlabel('time, t (msec)');
ylabel('current, i_L (A)');
figure(2); semilogy(t,i_L_1,'r-');
xlabel('time, t (msec)');
ylabel('current, i_L (A)');
t_sw = 35;% switching time in msec
tau2 = 16.67;
% determine array index for switching time
n_sw = find(t >= t_sw);
n_sw = n_sw(1);
%
i_L_2 = i_L_1(n_sw)*exp(-(t-t_sw)/tau2);
% derive composite signal from two components
i_L = (0 <=t & t <= t_sw).*i_L_1 + (t_sw <=t & t <= tmax).*i_L_2;
figure(3);plot(t,i_L,'r-')
xlabel('time, t (msec)');
ylabel('current, i_L (A)');
figure(4);semilogy(t,i_L,'r-')
xlabel('time, t (msec)');
ylabel('current, i_L (A)');
```